PARAMETERIZED TOPOLOGIES FOR IMPROVED NON-PID CONTROLLERS

PREPARATORY STEPS FOR PARAMETERIZED TOPOLOGIES FOR IMPROVED NON-PID CONTROLLER

TERMINAL SET

- There are 4 free variables representing the overall characteristics of a plant.
 - plant's ultimate gain, K_u (TU);
 - ultimate period, T_u (TU);
 - dead time, L (L);
 - time constant, T_r (TR)

• A constrained syntactic structure enforces the use of one terminal set for the arithmeticperforming subtrees and another terminal set for all other parts of the program tree.

• Arithmetic-performing subtrees may appear in both the result-producing branch and automatically defined functions (if any) that are created during the run by the architecture-altering operations.

TERMINAL SET—CONTINUED

• The numerical parameter value for each signal-processing block possessing a parameter is established by an arithmetic-performing subtree that may contain

- perturbable numerical values
- arithmetic operations
- the free variables (KU, TU, L, TR)
- The terminal set for the arithmeticperforming subtrees is

 $\mathbf{T}_{aps} = \{\Re, KU, TU, L, TR\}$

• R denotes a perturbable numerical value between -3.0 and +3.0

• The value returned by an entire arithmetic-performing subtree is interpreted by a nonlinear mapping as a component value lying in a range of (positive) values between 10⁻³ and 10³.

BOOTSTRAPPING ON THE 1995 ÅSTRÖM-HÄGGLUND TUNING RULES

• A terminal (called AH) is inserted into the terminal set representing the time-domain output of a PID controller that is tuned for the plant under consideration according to the 1995 Åström and Hägglund tuning rules

• That is, the AH terminal causes a connection to be made to the output port of a PID controller that was tuned using the 1995 Åström-Hägglund rules for the particular instantiations (for the plant under consideration) of the free variables for ultimate gain, K_u ; ultimate period, T_u ; dead time, L; and time constant, T_r .

PREPARATORY STEPS—CONTINUED

TERMINAL SET—CONTINUED

• The terminal set, **T**, for all other parts of the result-producing branch and any automatically defined functions contains are time-domain signals:

T = {AH, REFERENCE_SIGNAL, CONTROLLER_OUTPUT, PLANT_OUTPUT, LEFT_1, ..., LEFT_4, RIGHT_1, ..., RIGHT_4}

• LEFT_1, ..., LEFT_4, RIGHT_1, ..., RIGHT 4 are explained below

FUNCTION SET—CONTINUED

- A constrained syntactic structure enforces the use of one function set for the arithmeticperforming subtrees and another function set for all other parts of the program tree.
- The function set, F_{aps} , for the arithmeticperforming subtrees is

 $F_{aps} = \{+, -, *, \%, REXP, RLOG, POW\}.$

• These functions are protected in the sense that if the absolute value returned by any of these functions (unless 0) is less than 10⁻⁹ or greater than 10⁹, a value of 10⁻⁹ or 10⁹, respectively, is instead returned.

FUNCTION SET—CONTINUED

• The function set, F, for all other parts of the result-producing branch and automatically defined functions (if any) is $F = \{GAIN, INVERTER, LEAD, LAG, LAG2, DIFFERENTIAL_INPUT_INTEGRATOR, DIFFERENTIATOR, ADD_SIGNAL, SUB_SIGNAL, ADD_3_SIGNAL, MUL_SIGNAL, DIV_SIGNAL, TAKEOFF, ADF0, ADF1, ADF2, ADF3, ADF4\}$

TAKEOFF FUNCTION

• To create takeoff points (which are <u>common</u> in controllers), we use TAKEOFF function (instead of the <u>rarely invoked</u> architecture-altering operations)

• The one-argument **TAKEOFF** function acts as an identity function in that it returns the value of its argument.

- The TAKEOFF function stores the value of its argument so as to make this value potentially available to other points in the block diagram.
- If a subsequent *takeoff point reference terminal* appears in the program tree, the effect is to connect points in the controller's block diagram.

TAKEOFF FUNCTION—CONTINUED

• There are 8 takeoff point reference terminals (LEFT_1, ..., LEFT_4 and RIGHT_1, ..., RIGHT 4)

• Whenever LEFT 1, ..., LEFT 4 is encountered, it returns the value stored by the 1st, 2nd, 3rd, or 4th TAKEOFF function (if any), respectively, occurring earlier (i.e., to the left) in the overall program tree.

- For this purpose, the earlier TAKEOFF function is determined on the basis of the usual depth-first order of evaluation (from left to right) used in the LISP programming language.
- If there is no such stored value, the terminal defaults to the value of the root of the tree (i.e., the controller's output).
- Same idea for RIGHT_1, ..., RIGHT_4

PROGRAM ARCHITECTURE

• Because of use of TAKEOFF function, the architecture of each to-be-evolved program is a single result-producing branch (i.e., the architecture-altering operations are not used).

FITNESS MEASURE

• When there are free variables in a problem, it is necessary to ascertain the behavior and characteristics of each candidate individual for a representative sample of values for each of the free variables.

- Multiple combinations of values of the free variables force generalization of the to-be-evolved controller.
- Generalization is achieved in this problem by considering 6 different plants.
- The fitness measure for this problem aims to
 - optimize the integral of the time-weighted absolute error for a step input
 - optimize disturbance rejection
 - impose constraints on maximum sensitivity and sensor noise attenuation.

ÅSTRÖM AND HÄGGLUND'S 4 FAMILIES OF PLANTS

• Plants represented by transfer functions of the form

 $G(s) = \frac{e^{-s}}{(1+sT)^2}, \qquad [A]$

where *T*=0.1, ..., 10

• *n*-lag plants

 $G(s) = \frac{1}{(1+s)^n} , \qquad [B]$

where *n*=3, 4, and 8

• Plants represented by transfer functions of the form

 $G(s) = \frac{1}{(1+s)(1+\alpha s)(1+\alpha^2 s)(1+\alpha^3 s)},$ [C]

where α=0.2, 0.5, and 0.7

• Plants represented by transfer functions of the form

 $G(s) = \frac{1 - \alpha s}{(s+1)^3}, \qquad [D]$

where α=0.1, 0.2, 0.5, 1.0, and 2.0

TEST BED OF 33 PLANTS

Family	Parameter value	Ku	Tu	L	Tr	Runs in
						which the
						plant is used
А	T = 0.1	1.07	2.37	1.00	0.103	A, P, 1, 2, 3
А	T = 0.3	1.40	3.07	1.01	0.299	A, P, 1, 2, 3
А	T = 1	2.74	4.85	1.00	1.00	A, P, 1, 2, 3
А	T = 3	6.80	7.87	1.02	2.99	A, P, 1, 2, 3
А	T = 4.5	9.67	9.60	1.00	4.50	Р
A	T = 6	12.7	11.1	1.00	6.00	P, 2, 3
A	T = 7.5	15.6	12.3	1.00	7.50	Р
A	T = 9	18.7	13.4	1.01	9.00	Р
A	T = 10	20.8	14.2	0.916	10.1	A, P, 1, 2, 3
В	n = 3	8.08	3.62	0.517	1.24	A, P, 1, 2, 3
В	n = 4	4.04	6.27	1.13	1.44	A, P, 1, 2, 3
В	n = 5	2.95	8.62	1.79	1.61	P, 2, 3
В	n = 6	2.39	10.9	2.45	1.78	P, 1, 2, 3
В	n = 7	2.09	13.0	3.17	1.92	P, 2, 3
В	n = 8	1.89	15.2	3.88	2.06	A, P, 1, 2, 3
В	n = 11	1.57	21.6	6.19	2.41	1
С	$\alpha = 0.1$	113	0.198	-0.244	0.674	1
С	$\alpha = 0.2$	30.8	0.562	-0.137	0.691	A, P, 1, 2, 3
С	$\alpha = 0.215$	26.6	0.626	-0.155	0.713	Р
С	$\alpha = 0.23$	23.6	0.693	-0.116	0.705	Р
С	$\alpha = 0.26$	19.0	0.833	-0.099	0.722	Р
С	$\alpha = 0.3$	15.0	1.04	-0.024	0.720	P, 2, 3
С	$\alpha = 0.4$	9.62	1.59	0.111	0.759	P, 2, 3
С	$\alpha = 0.5$	6.85	2.23	0.267	0.804	A, P, 1, 2, 3
С	$\alpha = 0.6$	5.41	2.92	0.431	0.872	P, 2, 3
С	$\alpha = 0.7$	4.68	3.67	0.604	0.962	A, P, 1, 2, 3
С	$\alpha = 0.9$	4.18	5.31	3.44	0.685	1
D	$\alpha = 0.1$	6.21	4.06	0.644	1.22	A, P, 1, 2, 3
D	$\alpha = 0.2$	5.03	4.44	0.739	1.23	A, P, 1, 2, 3
D	$\alpha = 0.5$	3.23	5.35	1.15	1.17	A, P, 1, 2, 3
D	$\alpha = 0.7$	2.59	5.81	1.38	1.16	P, 2, 3
D	$\alpha = 1$	2.02	6.29	1.85	1.07	A, P, 1, 2, 3
D	$\alpha = 2$	1.15	7.46	3.46	0.765	A, P, 1, 2, 3

TEST BED OF 33 PLANTS USED IN 3 RUNS OF PROBLEM OF SYNTHESIZING PARAMETERIZED TOPOLOGIES FOR IMPROVED NON-PID CONTROLLERS

- 16 "A" plants used in Åström-Hägglund (1995)
- 20 "1" plants that are used to evolve the 1st non-PID controller
- 24 "2"/"3" plants used to evolve the 2nd/3rd non-PID controller

18 ADDITIONAL PLANTS

• All 18 additional plants are members of Åström and Hägglund's families A, C, and D

Family	Parameter value	K_u	T_{μ}	L	T_r
Α	0.15	1.13	2.57	0.993	0.153
А	0.5	1.74	3.65	0.982	0.509
А	0.9	2.51	4.60	1.011	0.894
А	2.5	5.69	7.25	0.999	2.50
А	4.0	8.68	9.07	1.002	4.00
А	9.0	18.7	13.4	1.005	9.00
С	0.25	20.3	0.786	-0.099	0.713
С	0.34	12.0	1.25	0.005	0.744
С	0.43	8.35	1.77	0.144	0.775
С	0.52	6.40	2.36	0.287	0.821
С	0.61	5.30	3.00	0.439	0.884
С	0.69	4.72	3.60	0.563	0.965
D	0.15	5.52	4.26	0.680	1.23
D	0.3	4.21	4.77	0.846	1.23
D	0.6	2.86	5.54	1.24	1.18
D	0.85	2.25	6.03	1.62	1.12
D	1.2	1.74	6.57	2.16	1.02
D	1.8	1.25	7.25	3.15	0.824

FITNESS MEASURE—CONTINUED

• For each plant under consideration, the fitness of each controller is measured by means of 8 separate invocations of the SPICE simulator. The fitness of an individual controller is the sum, over the 8 fitness cases for each plant under consideration, of the detrimental contributions to fitness. The smaller the sum, the better.

• Because there are 20 or 24 plants (depending on run), the fitness measure makes 160 or 192 separate invocations of the SPICE simulator.

PREPARATORY STEPS—CONTINUED

FITNESS MEASURE—CONTINUED

OVERALL SYSTEM CONTAINING A CONTROLLER AND PLANT WITH ADDITIVE DISTURBANCE SIGNAL AND SENSOR NOISE SIGNAL



FIRST 6 PARTS OF FITNESS MEASURE FOR EACH PLANT

- Step response and disturbance rejection are measured by means of the first 6 of the 8 SPICE simulations for each plant under consideration.
- Specifically, for each plant under consideration, 6 combinations of values for the height of the reference signal and disturbance signal are considered.
- The reference signal is a step function that rises from 0 at time *t*=0 to its specified height at *t*=1 millisecond.

• The disturbance signal (added to the controller's output before it reaches the plant) is a step function that rises from 0 at time $t=10T_u$ to its specified height at $t=10T_u+1$ milliseconds. The disturbance signal is.

Reference signal	Disturbance signal
1.0	1.0
10-3	10 ⁻³
-10 ⁻⁶	10 ⁻⁶
1.0	-0.6
-1.0	0.0

0.0

1.0

FIRST 6 PARTS OF FITNESS MEASURE FOR EACH PLANT —CONTINUED

• For each plant, a transient analysis is performed for each of the 6 combinations of height of reference and disturbance signals.

• The function *e*(*t*) is the difference (error) at time *t* between the plant output and the reference signal.

• The contribution to fitness for each of the 6 elements of the fitness measure associated with a plant is based on the sum of two integrals of time-weighted absolute error.



- The 1^{st} term (the integral running from time t=0 to $t=10T_u$) accounts for the controller's step response.
- The 2^{nd} term (the integral running from time $t=10T_u$ to $t=20T_u$) accounts for disturbance rejection.

PREPARATORY STEPS—CONTINUED FIRST 6 PARTS OF FITNESS MEASURE FOR EACH PLANT —CONTINUED



The factor *B* multiplies *e(t)* by the reciprocal of the amplitude of the reference signal associated with that fitness case (so that all reference signals are equally influential). *B* is zero if reference signal is 0.
The factor *C* multiplies of *e(t)* by the reciprocal of the amplitude of the disturbance signals. *C* is zero if disturbance signal is 0.

The ITAE component is such that, all other things being equal, a change in the time scale by a factor of F results in a change in ITAE by F². The division of the integral by T_u² is an attempt to eliminate this artifact of the time scale and equalize each plant's influence.
The contribution to fitness is multiplied by 20 if the element is greater than that for the Åström and Hägglund controller.

7TH PART OF FITNESS MEASURE FOR EACH PLANT

• The stability margin of each plant under consideration is measured by means of the 7th SPICE simulation.

• For each plant under consideration, an AC sweep is performed using the SPICE simulator from a minimum frequency of $1/(1,000T_u)$ to a maximum frequency of $1,000/T_u$ while holding the reference signal R(s) and the disturbance signal D(s) at zero.

• The maximum sensitivity, M_s , is a measure of the stability margin.

• It is desirable to minimize the maximum sensitivity (and therefore maximize the stability margin).

• The quantity $1/M_s$ is the minimum distance between the Nyquist plot and the point (-1, 0) and is the stability margin incorporating both gain and phase margin.

7TH PART OF FITNESS MEASURE FOR EACH PLANT—CONTINUED

- The maximum sensitivity is the maximum amplitude of Q(s).
- The contribution to fitness is 0 if $M_s < 1.5$; 2(M_s -1.5) for 1.5 $\leq M_s \leq 2.0$; and 20(M_s -2.0)+1 for $M_s > 2.0$.
- For each plant under consideration, the contribution to fitness is multiplied by 10 if the element is greater than that for the Åström and Hägglund controller.

8TH PART OF FITNESS MEASURE FOR EACH PLANT

• Sensor noise attenuation is measured by means of the 8th SPICE simulation for each plant under consideration.

• Achieving favorable sensor noise attenuation is often in direct conflict with the goal of achieving a rapid response to setpoint changes and rejection of plant disturbances. • For each plant under consideration, an AC sweep is performed using the SPICE simulator from a minimum frequency of $10/T_u$ to a maximum frequency of $1,000/T_u$ while holding the reference signal R(s) and the disturbance signal D(s) at zero.

• The attenuation of sensor noise N(s) is measured at the plant output Y(s). A_{min} is the minimum attenuation in decibels within this frequency range.

8TH PART OF FITNESS MEASURE FOR EACH PLANT—CONTINUED

• It is desirable to maximize the minimum attenuation.

• The contribution to fitness for sensor noise attenuation is 0 if $A_{min} > 40$ dB; $(40-A_{min})/10$ if $20 \text{ dB} \le A_{min} \le 40$ dB; and $2+(20-A_{min})$ if $A_{min} \le 20$ dB.

• For each plant under consideration, the contribution to fitness is multiplied by 10 if the element is greater than that for the Åström and Hägglund controller.

CONTROL PARAMETERS

• For difficult problems, we typically choose the population size such that genetic programming can execute a reasonably large number of generations within the amount of computer time we are willing to devote to the problem.

• Advance testing indicated that the time required to measure the fitness of an individual would exceed 2 minutes per individual.

• Our intent was to spend several weeks on the run

• We chose a small population size of 100 for each node on 1,000-Pentium Beowulf-style parallel computer (for a total of 100,000).

• Each generation would take a little over three hours and that there would therefore be about eight generations per day and about 56 generations per week.

RESULTS

PARAMETERIZED TOPOLOGIES FOR IMPROVED NON-PID CONTROLLERS

RESULTS—1ST RUN

• The best-of-run individual from the first run emerged in generation 88.

• The fitness measure for this first run entails 160 separate invocations of the SPICE simulator (i.e., eight simulations for each of 20 plants).

The average time to evaluate the fitness of an individual was 2 minutes and 10 seconds.
Because the population size is 100,000, the fitness of about 8,900,000 individuals was evaluated during the run.

• It took 320 hours (13.3 days) to produce the best-of-run individual.



Controller 301

RESULTS—1ST RUN—CONTINUED

• There are eight gain blocks (326, 327, 332, 334, 336, 382, 384, and 385) that are parameterized by a constant numerical amplification factor.

- Gain blocks 326, 332, 382, and 384 each have a gain of 3.
- Gain blocks 327, 334, 336, and 385 each have a gain of 2.

• There are four two-argument adders (324, 352, 388, and 387)

• There are seven two-argument subtractors (308, 312, 338, 346, 366, 376, and 386).

• There are two three-argument subtractors (328 and 335) in which one signal is subtracted from the sum of the two other signals.

• There are five gain blocks (310, 330, 340, 360, and 370) that are parameterized by genetically evolved non-constant mathematical expressions containing free variables.

RESULTS—1ST RUN—CONTINUED

• There is 1 Åström-Hägglund controller 306 (which contains additional adder, subtractor, integration, differentiation, and gain blocks).

RESULTS—1ST RUN—CONTINUED

• Gain block 310

 $10^{\log \log \log \left| \log \left(e^{K_u * L} \right) / L \right|}$ [11]

• Gain block 330:

$10^{e^{\log\left \log\left K_u*L\right \right }}$	[13]

• Gain block 340:

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e^{\log|K_u/L|} [14]
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• Gain block 360:

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10^{e^{\log |\log|K_u * L||}} [16]
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• Gain block 370:

 $e^{\log(K_u)}$ [17]

The first parameterized controller also has three lag blocks (320, 350, and 380) that are parameterized by genetically evolved nonconstant mathematical expression:

 T_r

RESULTS—1ST RUN—CONTINUED

INTERNAL FEEDBACK

• The first parameterized controller has internal feedback in several places.

- The final output 390 is fed back to seven other blocks (308, 312, 327, 336, 338, 346, and 384) within this controller.
- There is internal feedback involving subtractor 376, lag block 380, and adder 387 in which the signal that is fed back is not the controller's own final output. That is, this internal feedback loop lies <u>entirely inside</u> the controller.

RESULTS—1ST RUN—CONTINUED

• Genetic programming automatically created all the following:

- the controller's topology, including
 - the total number of processing blocks (30),
 - the type of each block,
 - all the connections (directed lines) that exist between the controller's two external input points, the controller's 30 blocks, and the controller's external output point,
- the controller's tuning, including
 - 3 mathematical expressions containing free variables (for the three lag blocks),
 - 5 mathematical expressions containing free variables (for 5 of the 13 gain blocks),
 - 8 constant mathematical expressions (for 8 of the 13 gain blocks).

RESULTS—1ST RUN—CONTINUED

• The best-of-run controller from generation 88 can be described in terms of its transfer function:

 $U = \frac{A(3+3G_{LAG}^{2}E_{13}E_{14}E_{16}E_{17}+G_{LAG}^{3}E_{11}E_{13}E_{14}E_{16}E_{17}+3G_{LAG}(1+E_{16}E_{17}))+(2+2G_{LAG}^{2}E_{13}E_{14}E_{16}E_{17}+G_{LAG}(-1+2E_{16})E_{17})R}{4+G_{LAG}^{3}(1+E_{11})E_{13}E_{14}E_{16}E_{17}+G_{LAG}^{2}(1+E_{14}+2E_{13}E_{14})E_{16}E_{17}+2G_{LAG}(2+E_{16}E_{17})},$

where

- U is the controller output,
- A is the output of the Åström-Hägglund controller,
- *R* is the reference signal,
- E_{11} through E_{17} correspond to equations 11 through 17, respectively, and
- G_{LAG} is the transfer function for the identical lag blocks, namely

 $G_{LAG} = \frac{1}{1+T_r*s}$

COMMON FEATURES OF ALL 3 RUNS

• First, each controller includes an Åström-Hägglund block as well as a second subcontroller that receives the output from the Åström-Hägglund controller.

• Second, the difference between the output of the Åström-Hägglund controller and output of the second sub-controller is computed at least once.

• Third, each second sub-controller makes use of internal feedback in the sense that it feeds its own output back into itself.

• Fourth, each second sub-controller includes at least one lag block or at least one lead block (in addition to the usual gain, integrative, derivative, additive, and subtractive blocks).

RESULTS—1ST **RUN**—CONTINUED

• The best-of-run controller from generation 88 is an improvement over the PID controller developed by Åström and Hägglund in their 1995 book.

• Averaged over the 20 plants used in this run, the first parameterized controller has

- 66.4% of the setpoint ITAE of the Åström-Hägglund controller,
- 85.7% of the disturbance rejection ITAE of the Åström-Hägglund controller,
- 94.6% of the reciprocal of minimum attenuation of the Åström-Hägglund controller, and
- 92.9% of the maximum sensitivity, *M_s*, of the Åström-Hägglund controller.

RESULTS—1ST RUN—CONTINUED

• Averaged over the 18 additional plants, the first parameterized controller has

- 64.1% of the setpoint ITAE of the Åström-Hägglund controller,
- 84.9% of the disturbance rejection ITAE of the Åström-Hägglund controller,
- 95.8% of the reciprocal of minimum attenuation of the Åström-Hägglund controller, and
- 93.5% of the maximum sensitivity, *M_s*, of the Åström-Hägglund controller.

RESULTS—1ST RUN—CONTINUED

COMPARISON OF CUMULATIVE ITAE FOR THE BEST-OF-RUN PARAMETERIZED CONTROLLER FROM THE FIRST RUN AND THE ÅSTRÖM-HÄGGLUND CONTROLLER FOR THE THREE-LAG PLANT



• The genetically evolved controller virtually eliminates the error at about 6 seconds whereas the Åström and Hägglund controller continues to accumulate error

RESULTS—1ST RUN—CONTINUED

PERCENTAGE COMPARISON (FOR 20 PLANTS) OF THE BEST-OF-RUN CONTROLLER FROM THE FIRST RUN AND THE ÅSTRÖM-HÄGGLUND CONTROLLER

Plant	Parameter	ITAE	ITAE 2	ITAE 3	ITAE 4	ITAE 5	ITAE 6	Stability	Sensor
	value	1						margin	noise
Α	0.1	86.8	87	86.1	87	77.8	90.8	59.4	84.2
А	0.3	86.3	86.3	86	86.3	64.1	99.9	70.5	88.3
А	1	78.9	78.9	78.8	74.7	60.6	88.2	89.4	76.9
А	3	67.8	67.8	67.6	67.8	62.4	79.3	71.7	73.3
А	10	73.1	73.3	73.1	73	72	78.3	0	87.6
В	3	67.7	67.8	67.9	67.7	64.8	74.3	0	OK
В	4	69.7	69.7	69.9	69.7	62.8	78	47.5	OK
В	6	78.4	78.5	78.5	78.4	70	85.4	78.9	OK
В	7	80.1	80.2	80.2	80.2	69	88	80.2	OK
В	8	83.3	83.3	83.2	83.2	70.9	91.7	79.5	OK
С	0.1	77.4	78.2	78.5	77.4	77	90.4	OK	30.9
С	0.2	76	76	75.8	76	74.7	86.2	OK	0
С	0.5	70.5	70.4	70.3	70.5	64.7	80.7	32.8	OK
С	0.7	68.3	68.3	68.6	68.3	61.6	77.2	42.7	OK
С	0.9	69.2	69.1	69	69.1	61.9	77.1	47.3	OK
D	0.1	67.1	67.3	67.3	66.8	62.2	74.7	14.2	OK
D	0.2	67.4	67.3	67.1	67.4	61.3	76	32.8	OK
D	0.5	70.3	70.4	70.3	70.4	59.2	80.7	64.2	OK
D	1	73.9	73.6	73.7	73.9	60.4	84.4	70.9	OK
D	2	83.3	83.2	83.4	83.1	74.9	88.2	61.5	OK

RESULTS—1ST RUN—CONTINUED

PERCENTAGE COMPARISON (ON THE 18 ADDITIONAL PLANTS) OF THE BEST-OF-RUN CONTROLLER FROM THE FIRST RUN AND THE ÅSTRÖM-HÄGGLUND CONTROLLER

				-			-		
Plant	Parame	ITAE 1	ITAE 2	ITAE 3	ITAE 4	ITAE 5	ITAE 6	Stabilit	Sensor
	ter							у	noise
	value							margin	
А	0.15	90	90	89.4	89.9	77.8	97.2	63.5	41.2
А	0.5	88.6	88.6	88.5	88.6	62.5	96.4	72.4	71.4
А	0.9	76.8	76.8	72.2	72.6	58.2	85.7	91	91.6
Α	2.5	66.3	66.3	66.2	66.4	60.2	77.8	73.6	75.6
А	4.0	68.9	68.9	68.8	68.8	65.8	77.2	70.2	100.3
А	9.0	71.2	71.3	71.1	71	70	76.5	0	87.8
С	0.25	72.9	72.8	72.8	72.9	71.3	82.1	235.9	OK
С	0.34	69.5	69.2	69.4	69.5	67	77.8	3.9	OK
С	0.43	70.8	70.8	70.8	70.5	66	81.2	37.1	OK
С	0.52	68.1	68.1	67.9	68.1	62.6	77.1	34.3	OK
С	0.61	68.4	68.5	68.3	68.4	62.4	77	37.7	OK
С	0.69	68.6	68.6	69	68.6	61.9	77.3	41.9	OK
D	0.15	67.1	67.1	67.2	67.1	61.9	74.8	25.2	OK
D	0.3	67	67	67	67	59.2	76.4	45.2	OK
D	0.6	67.9	67.8	67.9	67.9	56.4	78.3	67.4	OK
D	0.85	70	69.8	69.8	70.1	58.6	79.3	73.7	OK
D	1.2	75.4	75.3	75.2	75.3	61.3	84.7	70.7	OK
D	1.8	80.6	80.6	80.6	80.8	70.5	86.9	33.1	OK

PARSIMONY—2ND RUN

• Most controllers (nowadays) are programmed on digital microprocessors.

• Hence, parsimony is (usually) somewhat less important in controllers (than, say, circuits).

• However, the microprocessors are often inexpensive (e.g., eight-bit chips) and have very limited processing capability (making parsimony can be important, particularly when the transfer function cannot be represented by a low-order polynomial or when the sampling rate is very high.

• The fitness measure for the 2nd run contains a 193rd element that considers the controller's parsimony.

• The parsimony element of the fitness measure is dominant only after a controller outperforms the Åström-Hägglund PID controller for all 192 SPICE simulations.

RESULTS—2ND RUN

• The best-of-run individual from the second run emerged in generation 38.

• The fitness measure for the second run entails 192 separate invocations of the SPICE simulator (i.e., 8 simulations for each of 24 plants).

• The average time to evaluate the fitness of an individual is about 2 minutes and 36 seconds.

• It took 397 hours (16.5 days) to produce the best-of-run individual.

RESULTS—2ND RUN

• Unfortunately, within a few hours after the 2nd run evolved its first individual scoring 192 hits, a citywide power failure (lasting longer than the 15 minutes covered by our uninterruptable power supply) prematurely terminated this 16-day run.

• Parsimony never came into play

• Thus, although the second controller outperforms the Åström-Hägglund controller (by a small margin), it is not particularly parsimonious.

• Nonetheless, this second controller is noteworthy because it is another topologically different controller that outperforms the Åström-Hägglund PID controller and because it helps establish the common features of the 3 solutions.



RESULTS—2ND RUN—CONTINUED

• Lag block 410:

$$\log |2T_r + K_u^L|$$
. [21]

• Lag block 420:

$$\log \left| T_r + K_u^{(\frac{0.68631}{\frac{1}{K_u^{INT_1}} - T_r^L - T_u})} \right| \quad [22]$$

where



• Lag block 430:

$$\log \left| 2T_r + K_u^{\log \left| T_r + K_u^L \right|} \right| \quad \begin{bmatrix} 23 \end{bmatrix}$$

• Lag block 440:

$$\log |T_r + K_u^L|$$
 [24]

• Lag block 450:

$$|\log|T_r + \log(T_r + 1.2784)||$$
 [25]

RESULTS—2ND RUN—CONTINUED

• Lag block 460:

$$\log \left| T_r + \left(T_r + (x)^{\log \left| \log \left| K_u \wedge L \right| \right|} \right)^L \right| \quad [26]$$

where

$$x = T_r + K_u^{\log \left| \log \left| T_r + K_u^L \right| \right|}$$

• Lag block 470:

$$\log \left| T_r + \left(T_r + K_u^L \right)^L \right| \quad \begin{bmatrix} 27 \end{bmatrix}$$

Lag block 480:

$$\log |2T_r + K_u^L|$$
 [28]

RESULTS—2ND RUN—CONTINUED

• The best-of-run parameterized controller from generation 38 can be described in terms of its transfer function

U =

$$-\frac{-3R+A\left(-15-2/\left(1+E_{25}*s\right)-2/\left(1+E_{28}*s\right)\right)}{16+\frac{1}{1+E_{21}*s}-\frac{1}{1+E_{22}*s}+\frac{1}{1+E_{23}*s}+\frac{1}{1+E_{24}*s}+\frac{1}{\left(1+E_{25}*s\right)\left(1+E_{26}*s\right)}+\frac{1}{\left(1+E_{27}*s\right)\left(1+E_{28}*s\right)}}/\frac{1}{1+E_{28}*s}$$

RESULTS—2ND RUN—CONTINUED

• The best-of-run parameterized controller from generation 38 is an improvement over the PID controller developed by Åström and Hägglund in their 1995 book.

• Averaged over the 24 plants used in this run, the second controller has

- 85.5% of the setpoint ITAE of the Åström-Hägglund controller,
- 91.8% of the disturbance rejection ITAE of the Åström-Hägglund controller,
- 98.9% of the reciprocal of minimum attenuation of the Åström-Hägglund controller, and
- 97.5% of the maximum sensitivity, M_s , of the Åström-Hägglund controller.

RESULTS—2ND RUN—CONTINUED

Averaged over the 18 additional plants, the second controller has

- 84% of the setpoint ITAE of the Åström-Hägglund controller,
- 90.6% of the disturbance rejection ITAE of the Åström-Hägglund controller,
- 98.9% of the reciprocal of minimum attenuation of the Åström-Hägglund controller, and
- 97.5% of the maximum sensitivity, *M_s*, of the Åström-Hägglund controller.

RESULTS—3RD RUN

• The best-of-run controller emerged in generation 192.

• The fitness measure entails 192 separate invocations of the SPICE simulator (i.e., 8 simulations for each of 24 plants).

• The average time to evaluate the fitness of an individual was about 2 minutes and 5 seconds.

• It took 692 hours (28.8 days) to produce the best-of-run individual from generation 199. •

• This is the longest run of genetic programming that we have ever made.

RESULTS—3RD RUN—CONTINUED

• The first best-of-generation controller with 192 hits emerged in generation 105 after 422 hours (about 61% through the full 28.8-day run).

• The remaining 39% of the run was concerned with parsimony(weighted block count).

• The horizontal axis is the time (in hours) starting at 422 hours (17.6 days). The parsimony element at generation 105 is 22.6 (and at generation 199 is 13.1)





RESULTS—3RD RUN—CONTINUED

• The third controller is composed of leads (with transfer functions of the form $1+\tau s$) as well as gain, differentiator, integrator, adder, and subtractor blocks.

• Block 706 is a PID controller tuned with the Åström-Hägglund tuning rules for the plant under consideration.

• The third controller has four gain blocks (710, 720, 770, and 780) that are parameterized with a constant numerical amplification factor.

- Gain block 710 has a gain of 3.
- Gain blocks 720 and 770 have a gain of 2.
- Gain block 780 has a gain of 10.
- There are three three-argument adders (738, 748, and 788)
- There are three two-argument subtractors (734, 736, and 778).

RESULTS—3RD RUN—CONTINUED

• There are two gain blocks (730 and 760) whose gain is expressed as an equation involving the four free variables that describe a particular plant (i.e., the plant's ultimate gain, K_u ; ultimate period, T_u ; dead time, L; and time constant, T_r).

• Gain block 730 has a gain of

$$\log \left| T_r - T_u + \log \left| \frac{\log(|L|^L)}{T_u + 1} \right|$$
 [31]

• Gain block 760 has a gain of $|\log|_{T_r+1}||$ [34]

RESULTS—3RD RUN—CONTINUED

• There are two lead blocks (i.e., blocks with transfer functions of the form $1+\tau s$) that are parameterized by genetically evolved mathematical expressions.

• Lead block 740:

 $NLM\left(\log|L| - \left(\operatorname{abs}(L)^{L}\right)^{2} T_{u}^{3} \left(T_{u} + 1\right) T_{r} e^{L} - 2T_{u} e^{L}\right) \quad \begin{bmatrix} 32 \end{bmatrix}$

where *NLM* is the nonlinear mapping.Lead block 750:

 $NLM\left(\log|L| - 2T_u e^L \left(2K_u \left(\log \left| K_u e^L \right| - \log|L| \right) T_u + K_u e^L \right) \right) \quad \begin{bmatrix} 33 \end{bmatrix}$

RESULTS—3RD RUN—CONTINUED

• The best-of-run controller from generation 199 of the third run can be described in terms of its transfer function

 $U = \frac{R(1+3E_{34}(1+E_{32}*s)(1+E_{33}*s)) + A(10+E_{34}(3+E_{31}+2E_{32}*s+E_{31}E_{32}*s)(1+E_{33}*s))}{11+E_{34}(2+3E_{32}*s)(1+E_{33}*s)}$

RESULTS—3RD RUN—CONTINUED

• The best-of-run parameterized controller from generation 38 is an improvement over the PID controller developed by Åström and Hägglund in their 1995 book.

• Averaged over the 24 plants used in this run, the third controller has

- 81.8% of the setpoint ITAE of the Åström-Hägglund controller,
- 93.8% of the disturbance rejection ITAE of the Åström-Hägglund controller,
- 98.8% of the reciprocal of minimum attenuation of the Åström-Hägglund controller, and
- 93.4% of the maximum sensitivity, *M_s*, of the Åström-Hägglund controller.

RESULTS—3RD RUN—CONTINUED

• Averaged over the 18 additional plants, the third controller has

- 81.8% of the setpoint ITAE of the Åström-Hägglund controller,
- 94.2% of the disturbance rejection ITAE of the Åström-Hägglund controller,
- 99.7% of the reciprocal of minimum attenuation of the Åström-Hägglund controller, and
- 92.5% of the maximum sensitivity, *M_s*, of the Åström-Hägglund controller.