AUTOMATIC SYNTHESIS OF IMPROVED TUNING RULES FOR PID CONTROLLERS

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- **The PID controller was patented in 1939 by Albert Callender and Allan Stevenson of Imperial Chemical Limited of Northwich, England.**
- **The PID controller was an enormous improvement over previous manual and automatic methods for control.**
- **The quality of PID tuning rules is of considerable practical importance because a small percentage improvement in the operation of a plant can translate into large economic savings or other (e.g., environmental) benefits.**

IMPROVED TUNING RULES — CONTINUED

- **In 1942, Ziegler and Nichols developed a set of mathematical rules for automatically selecting the parameter values associated with the proportional, integrative, and derivative blocks of a PID controller.**
  - **The 1942 Ziegler-Nichols PID tuning rules do not require an analytic model of the plant.**
  - **Instead, they are based on several parameters that provide a simple characterization of the to-be-controlled plant.**
  - **These parameters have the practical advantage of being measurable for plants in the real world by means of relatively straightforward testing in the field.**
  - **The Ziegler-Nichols rules have been in widespread use for tuning PID controllers since World War II.**

IMPROVED TUNING RULES — CONTINUED

- **The question arises as to whether it is possible to improve upon the Ziegler-Nichols tuning rules.**
- **Åström and Hägglund answered that question in the affirmative in their 1995 book *PID Controllers: Theory, Design, and Tuning* in which they identified 4 families of plants**

> **"that are representative for the dynamics of typical industrial processes."**

ÅSTRÖM AND HÄGGLUND'S 4 FAMILIES OF PLANTS

- **Plants represented by transfer functions of the form**

$$G(s) = \frac{e^{-s}}{(1+sT)^2}, \qquad [A]$$

where *T*=0.1, … , 10

- ***n*-lag plants**

$$G(s) = \frac{1}{(1+s)^n}, \qquad [B]$$

where *n*=3, 4, and 8

- **Plants represented by transfer functions of the form**

$$G(s) = \frac{1}{(1+s)(1+\alpha s)(1+\alpha^2 s)(1+\alpha^3 s)}, \qquad [C]$$

where α=0.2, 0.5, and 0.7

- **Plants represented by transfer functions of the form**

$$G(s) = \frac{1-\alpha s}{(s+1)^3}, \qquad [D]$$

where α=0.1, 0.2, 0.5, 1.0, and 2.0

ÅSTRÖM AND HÄGGLUND'S APPROACH

- **Åström and Hägglund (1995) developed rules for automatically tuning PID controllers for the 16 plants from these 4 industrially representative families of plants.**
- **The PID tuning rules developed by Åström and Hägglund (like those of Ziegler and Nichols) are based on several parameters representing important overall characteristics of the plant that can be obtained by straightforward testing in the field.**
- **In one version of their method, Åström and Hägglund characterize a plant by two frequency-domain parameters:**
  - **ultimate gain, K_u (the minimum value of the gain that must be introduced into the feedback path to cause the system to oscillate)**
  - **ultimate period, T_u (the period of this lowest frequency oscillation)**

ÅSTRÖM AND HÄGGLUND'S APPROACH — CONTINUED

- **In another version of their method, Åström and Hägglund characterize a plant by two time-domain parameters**
  - **time constant, T_r**
  - **dead time, L (the period before the plant output begins to respond significantly to a new reference signal).**
- **Åström and Hägglund (1995) describe a procedure for estimating these two time-domain parameters from the plant's response to a simple step input.**

ÅSTRÖM AND HÄGGLUND'S APPROACH — CONTINUED

- **Åström and Hägglund (1995) set out to develop new tuning rules to yield improved performance with respect to the multiple (often conflicting) issues associated with most practical control systems including**
  - **setpoint response,**
  - **disturbance rejection,**
  - **sensor noise attenuation, and**
  - **robustness in the face of plant model changes as expressed by the stability margin.**
- **Åström and Hägglund approached the challenge of improving on the 1942 Ziegler-Nichols tuning rules with a shrewd combination of mathematical analysis, domain-specific knowledge, rough-and-ready approximations, creative flair, and intuition sharpened over years of practical experience.**

ÅSTRÖM AND HÄGGLUND'S APPROACH — CONTINUED

- **Åström and Hägglund started by identifying industrially representative analytic plant models**
- **Next, they characterized each of the plants in their test bed in terms of parameters that are easily measured in the field.**
- **Then they applied the known analytic design technique of dominant pole design to the analytic plant models and recorded the resulting parameters for PID controllers**
- **They decided that functions of the form**

$$f(x) = a_0 * e^{a_1 x + a_2 x^2},$$

where x=1/K_u, were the appropriate form for the tuning rules.

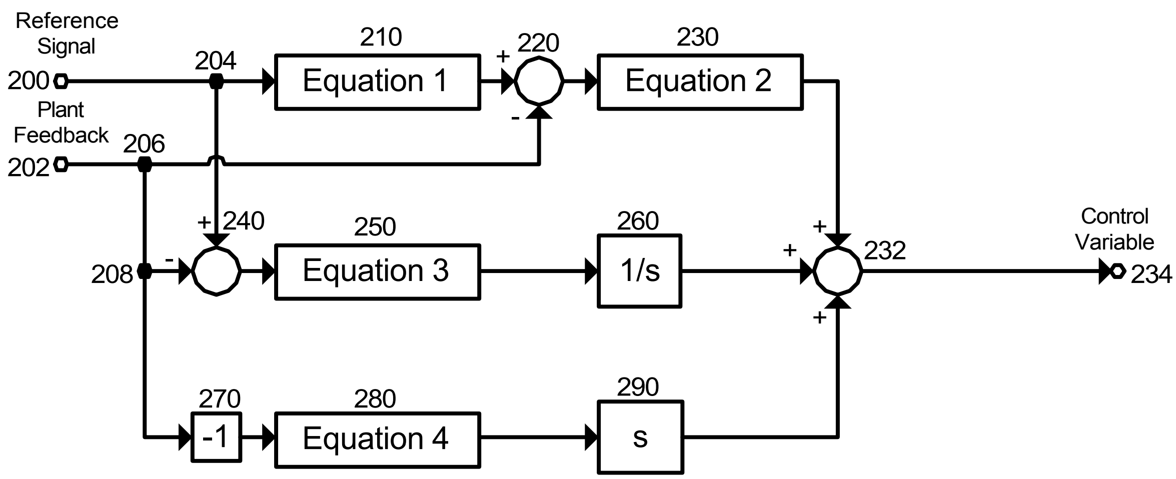
- **Finally, they fit approximating functions of the chosen form to the PID controller parameters produced by the dominant pole design technique.**

ÅSTRÖM AND HÄGGLUND (1995) RESULT

- **The tuning rules developed by Åström and Hägglund in their 1995 book outperform the 1942 Ziegler-Nichols tuning rules on all 16 industrially representative plants used by Åström and Hägglund. As Åström and Hägglund observe,**

> **"[Our] new methods give substantial improvements in control performance while retaining much of the simplicity of the Ziegler-Nichols rules."**

THE TOPOLOGY OF A PID CONTROLLER WITH NONZERO SETPOINT WEIGHTING OF THE REFERENCE SIGNAL IN THE PROPORTIONAL BLOCK 270 BUT NO SETPOINT WEIGHTING FOR THE DERIVATIVE BLOCK 280



ÅSTRÖM AND HÄGGLUND (1995) TUNING RULES

- **The PID tuning rules developed by Åström and Hägglund are expressed by 4 equations of the form**

$$f(x) = a_0 * e^{a_1 x + a_2 x^2},$$

where x=1/K_u.

- **Equation 1 implements setpoint weighting 210 of the reference signal 200 (the setpoint) for the proportional (P) part Equation 1 specifies that the setpoint weighting, *b*, is given by**

$$b = 0.25 * e^{\frac{0.56}{K_u} + \frac{-0.12}{K_u^2}}. [1]$$

- **Equation 2 specifies the gain, K_p, for the proportional (P) block 230 of the controller**

$$K_p = 0.72 * K_u * e^{\frac{-1.6}{K_u} + \frac{1.2}{K_u^2}}. [2]$$

ÅSTRÖM AND HÄGGLUND (1995) TUNING RULES — CONTINUED

- **Equation 3 specifies the gain 250, K_i, that is associated with the input to the integrative (I) block 260**

$$K_i = \frac{0.72 * K_u * e^{\frac{-1.6}{K_u} + \frac{1.2}{K_u^2}}}{0.59 * T_u * e^{\frac{-1.3}{K_u} + \frac{0.38}{K_u^2}}} . [3]$$

- **Equation 4 specifies the gain 280, K_d, that is associated with the input to the derivative (D) block 290**

$$K_d = 0.108 * K_u * T_u * e^{\frac{-1.6}{K_u} + \frac{1.2}{K_u^2}} * e^{\frac{-1.4}{K_u} + \frac{0.56}{K_u^2}} . [4]$$

ÅSTRÖM AND HÄGGLUND (1995) TUNING RULES — CONTINUED

- **The transfer function for the Åström-Hägglund PID controller is**

$$A = K_p(b*R-P)+\frac{K_i}{s}(R-P)+K_d*s*(-P),$$

  - **where *A* is the output of the Åström-Hägglund controller,**
  - ***R* is the reference signal,**
  - ***P* is the plant output, and *b*, K_p, K_i, and K_d are given by the four equations above**

- **Note that only –*P* (as opposed to *R*–*P*) appears in the derivative term of this transfer function because Åström and Hägglund apply a setpoint weighting of zero to the reference signal *R* in the derivative block.**

IMPROVED TUNING RULES USING GP

- **The goal here is to use genetic programming is used to discover PID tuning rules that improve upon the tuning rules developed by Åström and Hägglund in their 1995 book.**
- **The topology of the to-be-evolved controller is not open-ended here, but, instead, fixed as the PID controller topology.**
- **That is, we are *not* searching here for a parameterized topology**

TEST BED OF 33 PLANTS

Family	Parameter value	Ku	Tu	L	Tr	Runs in which the plant is used
A	T = 0.1	1.07	2.37	1.00	0.103	A, P, 1, 2, 3
A	T = 0.3	1.40	3.07	1.01	0.299	A, P, 1, 2, 3
A	T = 1	2.74	4.85	1.00	1.00	A, P, 1, 2, 3
A	T = 3	6.80	7.87	1.02	2.99	A, P, 1, 2, 3
A	T = 4.5	9.67	9.60	1.00	4.50	P
A	T = 6	12.7	11.1	1.00	6.00	P, 2, 3
A	T = 7.5	15.6	12.3	1.00	7.50	P
A	T = 9	18.7	13.4	1.01	9.00	P
A	T = 10	20.8	14.2	0.916	10.1	A, P, 1, 2, 3
B	n = 3	8.08	3.62	0.517	1.24	A, P, 1, 2, 3
B	n = 4	4.04	6.27	1.13	1.44	A, P, 1, 2, 3
B	n = 5	2.95	8.62	1.79	1.61	P, 2, 3
B	n = 6	2.39	10.9	2.45	1.78	P, 1, 2, 3
B	n = 7	2.09	13.0	3.17	1.92	P, 2, 3
B	n = 8	1.89	15.2	3.88	2.06	A, P, 1, 2, 3
B	n = 11	1.57	21.6	6.19	2.41	1
C	α = 0.1	113	0.198	-0.244	0.674	1
C	α = 0.2	30.8	0.562	-0.137	0.691	A, P, 1, 2, 3
C	α = 0.215	26.6	0.626	-0.155	0.713	P
C	α = 0.23	23.6	0.693	-0.116	0.705	P
C	α = 0.26	19.0	0.833	-0.099	0.722	P
C	α = 0.3	15.0	1.04	-0.024	0.720	P, 2, 3
C	α = 0.4	9.62	1.59	0.111	0.759	P, 2, 3
C	α = 0.5	6.85	2.23	0.267	0.804	A, P, 1, 2, 3
C	α = 0.6	5.41	2.92	0.431	0.872	P, 2, 3
C	α = 0.7	4.68	3.67	0.604	0.962	A, P, 1, 2, 3
C	α = 0.9	4.18	5.31	3.44	0.685	1
D	α = 0.1	6.21	4.06	0.644	1.22	A, P, 1, 2, 3
D	α = 0.2	5.03	4.44	0.739	1.23	A, P, 1, 2, 3
D	α = 0.5	3.23	5.35	1.15	1.17	A, P, 1, 2, 3
D	α = 0.7	2.59	5.81	1.38	1.16	P, 2, 3
D	α = 1	2.02	6.29	1.85	1.07	A, P, 1, 2, 3
D	α = 2	1.15	7.46	3.46	0.765	A, P, 1, 2, 3

TEST BED OF 33 PLANTS

- **16 “A” plants used in Åström-Hägglund (1995)**
- **30 “P” plants that are used to evolve the improved PID tuning rules**
- **20 “1” plants that are used to evolve the 1st non-PID controller**
- **24 “2”/”3” plants used to evolve the 2nd/3rd non-PID controller**

18 ADDITIONAL PLANTS

- **All 18 additional plants are members of Åström and Hägglund's families A, C, and D**

Family	Parameter value	K_u	T_u	L	T_r
A	0.15	1.13	2.57	0.993	0.153
A	0.5	1.74	3.65	0.982	0.509
A	0.9	2.51	4.60	1.011	0.894
A	2.5	5.69	7.25	0.999	2.50
A	4.0	8.68	9.07	1.002	4.00
A	9.0	18.7	13.4	1.005	9.00
C	0.25	20.3	0.786	-0.099	0.713
C	0.34	12.0	1.25	0.005	0.744
C	0.43	8.35	1.77	0.144	0.775
C	0.52	6.40	2.36	0.287	0.821
C	0.61	5.30	3.00	0.439	0.884
C	0.69	4.72	3.60	0.563	0.965
D	0.15	5.52	4.26	0.680	1.23
D	0.3	4.21	4.77	0.846	1.23
D	0.6	2.86	5.54	1.24	1.18
D	0.85	2.25	6.03	1.62	1.12
D	1.2	1.74	6.57	2.16	1.02
D	1.8	1.25	7.25	3.15	0.824

PERFORMANCE OF THE ÅSTRÖM-HÄGGLUND PID CONTROLLER ON THE 33 PLANTS

Plant	Plant parameter value	ITAE 1	ITAE 2	ITAE 3	ITAE 4	ITAE 5	ITAE 6	Stability margin	Sensor noise
A	0.1	0.740	0.740	0.749	0.743	0.269	0.478	0.980	10.1
A	0.3	0.701	0.701	0.702	0.701	0.266	0.434	0.669	1.75
A	1	0.510	0.510	0.511	0.510	0.237	0.292	0.660	1.11
A	3	0.331	0.331	0.331	0.331	0.211	0.122	0.594	1.49
A	4.5	0.282	0.282	0.282	0.282	0.201	0.082	0.410	1.34
A	6	0.259	0.259	0.259	0.259	0.198	0.062	0.279	1.25
A	7.5	0.244	0.244	0.244	0.244	0.195	0.049	0.158	1.16
A	9	0.236	0.236	0.236	0.236	0.195	0.041	0.070	1.13
A	10	0.232	0.232	0.232	0.232	0.195	0.037	0.016	1.11
B	3	0.334	0.334	0.335	0.334	0.225	0.108	0.368	0
B	4	0.456	0.456	0.456	0.456	0.238	0.215	0.707	0
B	5	0.527	0.526	0.526	0.527	0.243	0.281	0.690	0
B	6	0.570	0.571	0.569	0.570	0.245	0.321	0.646	0
B	7	0.600	0.599	0.597	0.599	0.248	0.348	0.642	0
B	8	0.623	0.623	0.621	0.623	0.251	0.369	0.643	0
B	11	0.667	0.667	0.663	0.665	0.257	0.405	0.641	0
C	0.1	0.187	0.187	0.187	0.187	0.181	0.006	0	0.659
C	0.2	0.193	0.194	0.193	0.193	0.171	0.022	0	0.024
C	0.215	0.196	0.196	0.196	0.196	0.171	0.025	0.039	0
C	0.23	0.201	0.201	0.200	0.201	0.173	0.028	0.081	0
C	0.26	0.211	0.210	0.210	0.211	0.175	0.035	0.156	0
C	0.3	0.225	0.225	0.224	0.225	0.178	0.046	0.241	0
C	0.4	0.272	0.272	0.272	0.272	0.191	0.079	0.421	0
C	0.5	0.325	0.325	0.325	0.325	0.207	0.117	0.542	0
C	0.6	0.374	0.373	0.374	0.374	0.219	0.152	0.594	0
C	0.7	0.414	0.413	0.412	0.414	0.231	0.181	0.675	0
C	0.9	0.452	0.452	0.451	0.452	0.238	0.213	0.711	0
D	0.1	0.187	0.187	0.187	0.187	0.181	0.006	0.585	0
D	0.2	0.193	0.193	0.194	0.193	0.171	0.022	0.699	0
D	0.5	0.225	0.225	0.225	0.225	0.178	0.046	0.728	0
D	0.7	0.272	0.272	0.272	0.272	0.191	0.079	0.746	0
D	1	0.325	0.325	0.325	0.325	0.207	0.116	0.839	0
D	2	0.374	0.373	0.374	0.374	0.219	0.152	4.18	0

PERFORMANCE OF THE ÅSTRÖM-HÄGGLUND PID CONTROLLER ON THE 18 ADDITIONAL PLANTS

Plant	Plant parameter value	ITAE 1	ITAE 2	ITAE 3	ITAE 4	ITAE 5	ITAE 6	Stability margin	Sensor noise
A	0.15	0.740	0.740	0.746	0.743	0.269	0.478	0.860	4.72
A	0.5	0.701	0.701	0.702	0.701	0.266	0.434	0.621	0.89
A	0.9	0.510	0.510	0.511	0.51	0.237	0.292	0.638	1.24
A	2.5	0.331	0.331	0.331	0.331	0.211	0.122	0.657	1.05
A	4.0	0.259	0.259	0.259	0.259	0.198	0.062	0.459	1.44
A	9.0	0.232	0.232	0.232	0.232	0.195	0.037	0.070	1.13
C	0.25	0.335	0.334	0.333	0.335	0.225	0.108	0.128	0
C	0.34	0.457	0.457	0.455	0.457	0.239	0.215	0.297	0
C	0.43	0.527	0.526	0.526	0.527	0.243	0.282	0.471	0
C	0.52	0.570	0.571	0.569	0.570	0.245	0.321	0.546	0
C	0.61	0.600	0.599	0.597	0.599	0.248	0.348	0.600	0
C	0.69	0.623	0.623	0.621	0.623	0.251	0.369	0.665	0
D	0.15	0.187	0.187	0.187	0.187	0.181	0.006	0.585	0
D	0.3	0.193	0.193	0.194	0.193	0.171	0.022	0.699	0
D	0.6	0.225	0.225	0.225	0.225	0.178	0.046	0.728	0
D	0.85	0.272	0.272	0.272	0.272	0.191	0.079	0.746	0
D	1.2	0.325	0.325	0.325	0.325	0.207	0.116	0.839	0
D	1.8	0.374	0.373	0.374	0.374	0.219	0.152	4.18	0

SEARCH FOR IMPROVED PID TUNING RULES

- **The controller's topology is not subject to evolution, but instead, is prespecified to be the PID topology**
- **We are seeking four mathematical expressions (for K_p, K_i, K_d, and b).**
- **Each of the expressions may contain free variables representing the plant's ultimate gain, K_u, and the plant's ultimate period, T_u.**
- **Note that there is no search for, or evolution of, the topology here. The problem is simply a search for four surfaces in three-dimensional space.**

SEARCH FOR IMPROVED PID TUNING RULES — CONTINUED

- **Initial experimentation quickly confirmed the fact that the Åström and Hägglund tuning rules are highly effective.**
- **Starting from scratch, genetic programming readily evolved three-dimensional surfaces that closely match the Åström-Hägglund surfaces and that outperform the Åström-Hägglund tuning rules *on average*.**
- **However, these genetically evolved surfaces did not satisfy our goal, namely outperforming the Åström-Hägglund tuning rules for every plant in the test bed.**

SEARCH FOR IMPROVED PID TUNING RULES — CONTINUED

- **Thus, we decided to approach the problem of discovering improved tuning rules by building on the known and highly effective Åström-Hägglund results.**
- **Specifically, we use genetic programming to evolve four mathematical expressions (containing the free variables K_u and T_u) which, *when added to* the corresponding mathematical expressions developed by Åström and Hägglund, yield improved performance on every plant in the test bed.**
- **Although we anticipated that the resulting three-dimensional surfaces would be similar to the Åström-Hägglund surfaces (i.e., the magnitudes of the added numbers would be relatively small), this additive approach can, in fact, yield any surface (or, more precisely, any surface that can be represented by the generously large number of points that a single program tree may contain).**

PREPARATORY STEPS FOR IMPROVED PID TUNING RULES

PROGRAM ARCHITECTURE

- **The architecture of each program tree in the population has four result-producing branches (one associated with each of the problem's four independent variables, namely K_p, K_i, K_d, and b).**

TERMINAL SET

The terminal set, T, for each of the four result-producing branches is

T = {$\Re$, `KU`, `TU`},

where $\Re$ denotes a perturbable numerical value between –5.0 and +5.0.

FUNCTION SET

F = {`+`, `-`, `*`, `%`, `REXP`, `RLOG`, `POW`}.

The two-argument `POW` function returns the value of its first argument to the power of the value of its second argument.

PREPARATORY STEPS FOR IMPROVED PID TUNING RULES — CONTINUED

FITNESS

- **Fitness is measured by means of eight separate invocations of the SPICE simulator for each plant under consideration.**
- **When the aim is to automatically create a solution to a category of problems in the form of mathematical expressions containing free variable(s), the possibility of overfitting is especially salient. Overfitting occurs when an evolved solution performs well on the fitness cases that are incorporated in the fitness measure (i.e., in the training phase), but then performs poorly on previously unseen fitness cases.**

PREPARATORY STEPS FOR IMPROVED PID TUNING RULES — CONTINUED

FITNESS — CONTINUED

- **In this problem, four presumptively nonlinear and complicated functions mathematical expressions (each incorporating two free variables) must be evolved.**

- **In applying genetic programming to this program, we were especially concerned about overfitting because there are only 16 plants in the test bed used by Åström and Hägglund. Moreover, there are only three plants in two of the four families.**
- **GP must usually consider a multiplicity of data points in order to discover even a linear relationship. *A fortiori*, a multiplicity of data points is required when the underlying function is actually nonlinear (as is usually the case in non-trivial problems).**

PREPARATORY STEPS FOR IMPROVED PID TUNING RULES — CONTINUED

FITNESS — CONTINUED

- **Concerned about possible overfitting, we used the 30 plants marked "P" (in the table) so that the fitness measure entails 240 separate invocations of the SPICE simulator.**
- **The choice of 30 plants for this first run was governed primarily by considerations of computer time. Ideally, we would have used even more plants.**
- **We did not know in advance whether 30 plants would prove to be sufficient to enable genetic programming to unearth the complicated relationships inherent in this problem.**

PREPARATORY STEPS FOR IMPROVED PID TUNING RULES — CONTINUED

CONTROL PARAMETERS

- **The population size is 100,000**

RESULTS — IMPROVED PID TUNING RULES

- **The best-of-run individual emerged in generation 76, so the fitness of approximately 7,700,000 individuals was evaluated during the run. The average time to evaluate the fitness of an individual (i.e., to perform the 240 SPICE simulations) was 51 seconds. It took 106.8 hours (4.4 days) to produce the best-of-run individual from generation 76.**
- **The improved PID tuning rules are obtained by adding the genetically evolved adjustments below to the values of K_p, K_i, and K_d, and *b* developed by Åström and Hägglund in their 1995 book.**

RESULTS — IMPROVED PID TUNING RULES — CONTINUED

•**The quantity, $K_{p\text{-}adj}$, that is to be added to K_p for the proportional part of the controller is**

$$K_{p\text{-}adj} = -.0012340 * T_u - 6.1173 * 10^{-6}.$$

• **The quantity, $K_{i\text{-}adj}$, that is to be added to K_i for the integrative part is**

$$K_{i\text{-}adj} = .\ -.068525 * \frac{K_u}{T_u}$$

• **The quantity, $K_{d\text{-}adj}$, that is to be added to K_d for the derivative part is**

$$K_{d\text{-}adj} = -0.0026640\left(e^{T_u}\right)^{\log\left(1.6342^{\log K_u}\right)}.$$

• **The quantity, b_{adj}, that is to be added to b for the setpoint weighting of the reference signal in the proportional block is**

$$b_{adj} = \frac{K_u}{e^{K_u}}.$$

RESULTS — IMPROVED PID TUNING RULES — CONTINUED

• **In other words, after adding, the final values ($K_{p\text{-}final}$, $K_{i\text{-}final}$, $K_{d\text{-}final}$, and b_{final}) for the genetically evolved PID tuning rules are as follows:**

$$K_{p\text{-}final} = 0.72 * K_u * e^{\frac{-1.6}{K_u} + \frac{1.2}{K_u^2}} - .0012340 * T_u - 6.1173 * 10^{-6}$$

$$K_{i\text{-}final} = \frac{0.72 * K_u * e^{\frac{-1.6}{K_u} + \frac{1.2}{K_u^2}}}{0.59 * T_u * e^{\frac{-1.3}{K_u} + \frac{0.38}{K_u^2}}} - .068525 * \frac{K_u}{T_u}$$

$$K_{d\text{-}final} = 0.108 * K_u * T_u * e^{\frac{-1.6}{K_u} + \frac{1.2}{K_u^2}} * e^{\frac{-1.4}{K_u} + \frac{0.56}{K_u^2}} - 0.0026640\left(e^{T_u}\right)^{\log\left(1.6342^{\log K_u}\right)}$$

$$b_{final} = 0.25 * e^{\frac{0.56}{K_u} + \frac{-0.12}{K_u^2}} + \frac{K_u}{e^{K_u}}$$

• **The authors refer to these genetically evolved PID tuning rules as the Keane-Koza-Streeter (KKS) PID tuning rules.**

• **The automatically designed controller is parameterized (i.e., general) in the sense that it contains free variables (K_u and T_u) and thereby provides a solution to an entire category of problems (i.e., the control of all the plants in all the families).**

RESULTS — IMPROVED PID TUNING RULES — CONTINUED

- **Averaged over the 30 plants used in this run, the best-of-run tuning rules from generation 76 have**
  - **91.6% of the setpoint ITAE of the Åström-Hägglund tuning rules,**
  - **96.2% of the disturbance rejection ITAE of the Åström-Hägglund tuning rules,**
  - **99.5% of the reciprocal of minimum attenuation of the Åström-Hägglund tuning rules, and**
  - **98.6% of the maximum sensitivity, M_s, of the Åström-Hägglund tuning rules.**

IMPROVED PID TUNING RULES — CONTINUED

CROSS-VALIDATION

- **Averaged over the 18 additional plants, the best-of-run tuning rules from generation 76 have**
  - **89.7% of the setpoint ITAE of the Åström-Hägglund tuning rules,**
  - **95.6% of the disturbance rejection ITAE of the Åström-Hägglund tuning rules,**
  - **99.5% of the reciprocal of minimum attenuation of the Åström-Hägglund tuning rules, and**
  - **98.5% of the maximum sensitivity, M_s, of the Åström-Hägglund tuning rules.**
- **As can be seen, the results obtained for the 18 previously unseen additional plants are similar to those for the results for the plants used by the evolutionary process.**

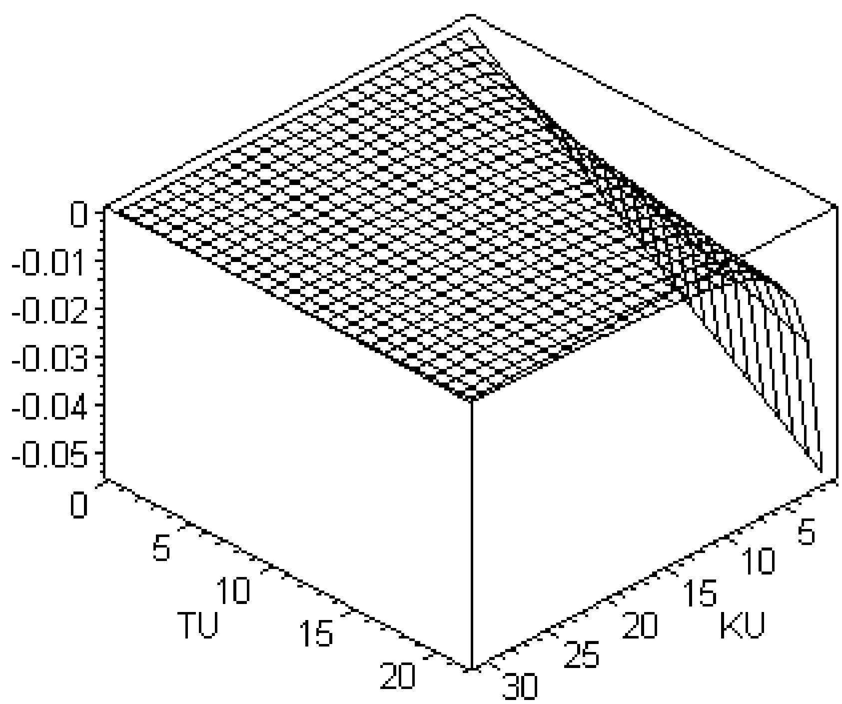
IMPROVED PID TUNING RULES — CONTINUED

COMPARISON TO ÅSTRÖM AND HÄGGLUND

- **Averaged over the 16 plants used by Åström and Hägglund in their 1995 book, the best-of-run tuning rules from generation 76 have**
  - **90.5% of the setpoint ITAE of the Åström-Hägglund tuning rules,**
  - **96% of the disturbance rejection ITAE of the Åström-Hägglund tuning rules,**
  - **99.3% of the reciprocal of minimum attenuation of the Åström-Hägglund tuning rules, and**
  - **98.5% of the maximum sensitivity, M_s, of the Åström-Hägglund tuning rules.**

COMPARISON TO ÅSTRÖM-HÄGGLUND TUNING RULES

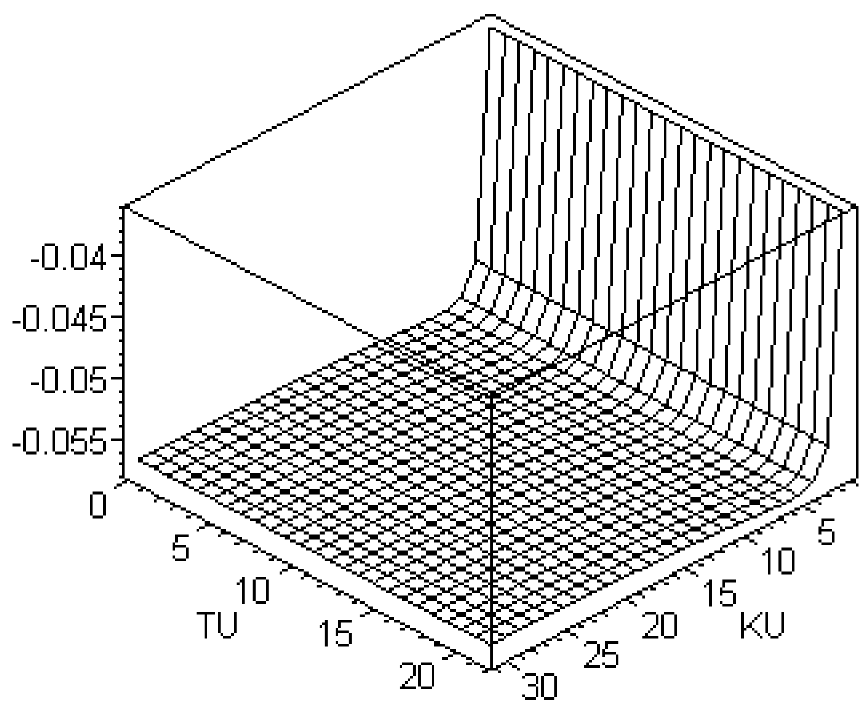
THE GENETICALLY EVOLVED ADJUSTMENT, $K_{P\text{-}ADJ}$, FOR THE PROPORTIONAL (P) PART



- **The genetically evolved adjustment becomes as large as –5% only in the combination of circumstances when K_u is at the low end of its range and T_u is at the high end of its range.**

COMPARISON TO ÅSTRÖM-HÄGGLUND TUNING RULES — CONTINUED

ADJUSTMENT, $K_{I\text{-}ADJ}$, FOR THE INTEGRATIVE (I) PART



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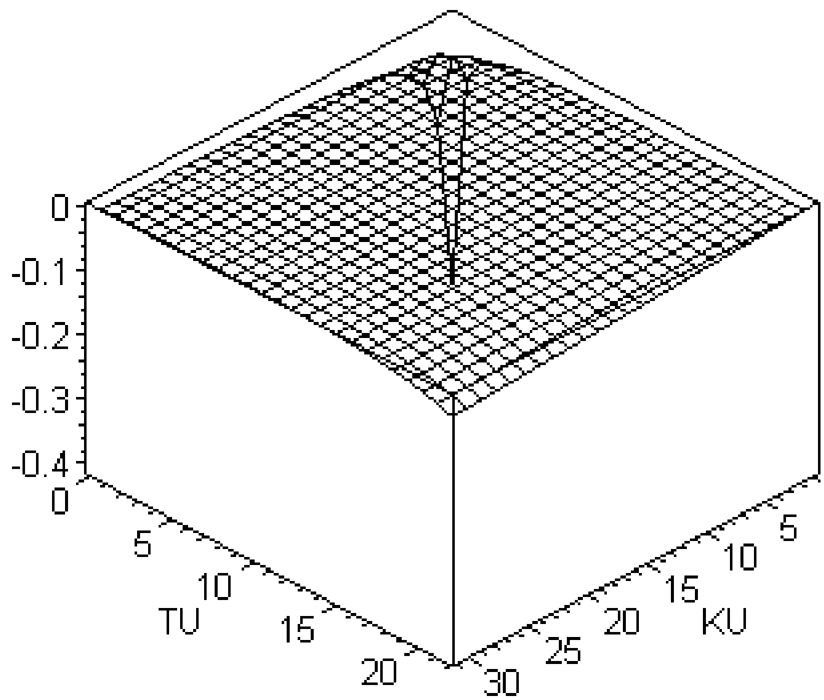
- **The genetically evolved adjustment is about –5.5% for most values of K_u, but drops to as little as –4% when K_u is at the low end of its range.**

COMPARISON — CONTINUED

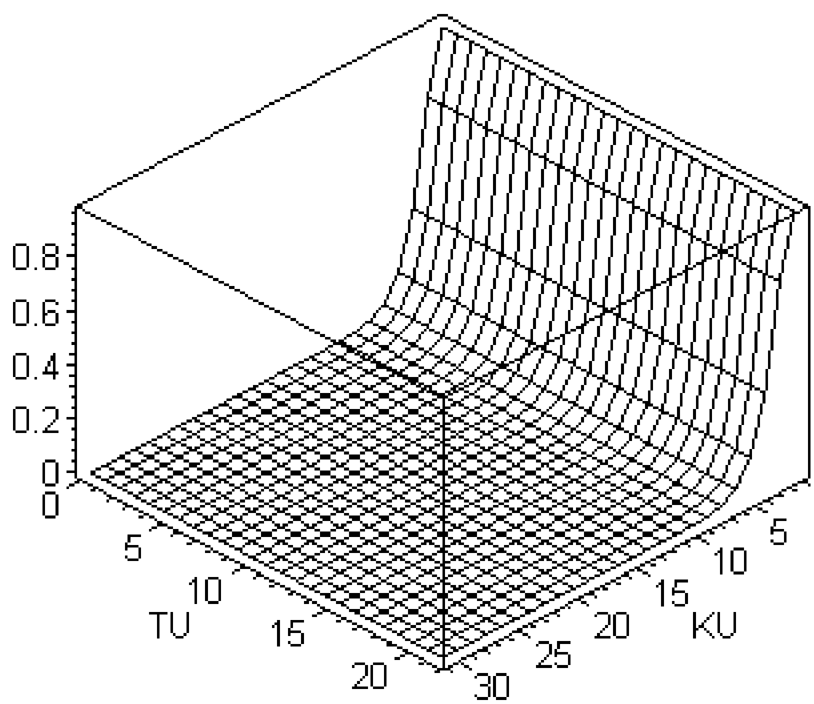
ADJUSTMENT, $K_{D\text{-}ADJ}$, FOR THE DERIVATIVE (D) PART



• As can be seen from the figure, the genetically evolved adjustment is near zero for most values of K_u and T_u ,but becomes as large as –40% when both K_u and T_u are at the low ends of their ranges.

COMPARISON — CONTINUED

ADJUSTMENT, B_{ADJ}, FOR THE SETPOINT WEIGHTING OF THE REFERENCE SIGNAL OF THE CONTROLLER'S P BLOCK



- **The genetically evolved adjustments range between 0% and about +80% and depend only on K_u. It is near zero for most values of K_u, but rises sharply when K_u is at the low end of its range.**