# AUTOMATIC SYNTHESIS OF CONTROLLERS

# CONTROLLERS

· **The purpose of a controller is to force, in a meritorious way, the plant's response to match a desired response (called the *reference signal* or *setpoint*).**

· **The output a controller is fed into the to-be-controlled system (conventionally called the *plant*).**

· **In closed loop controllers, the output of the plant is fed back (via external feedback) to the controller.**

· **The reference signal is typically compared to the plant output – typically by subtraction.**

· **The input of the controller is the reference signal and the plant output or, sometimes, just the difference between the two.**

# CONTROLLER DESIGN

**· The problem of synthesizing a controller to satisfy prespecified requirements is sometimes solvable by analytic techniques (often oriented toward producing conventional PID controllers).**

**· As Boyd and Barratt stated in *Linear Controller Design: Limits of Performance* (1991)**

> **"The challenge for controller design is to productively use the enormous computing power available. Many current methods of computer-aided controller design simply automate procedures developed in the 1930's through the 1950's …"**

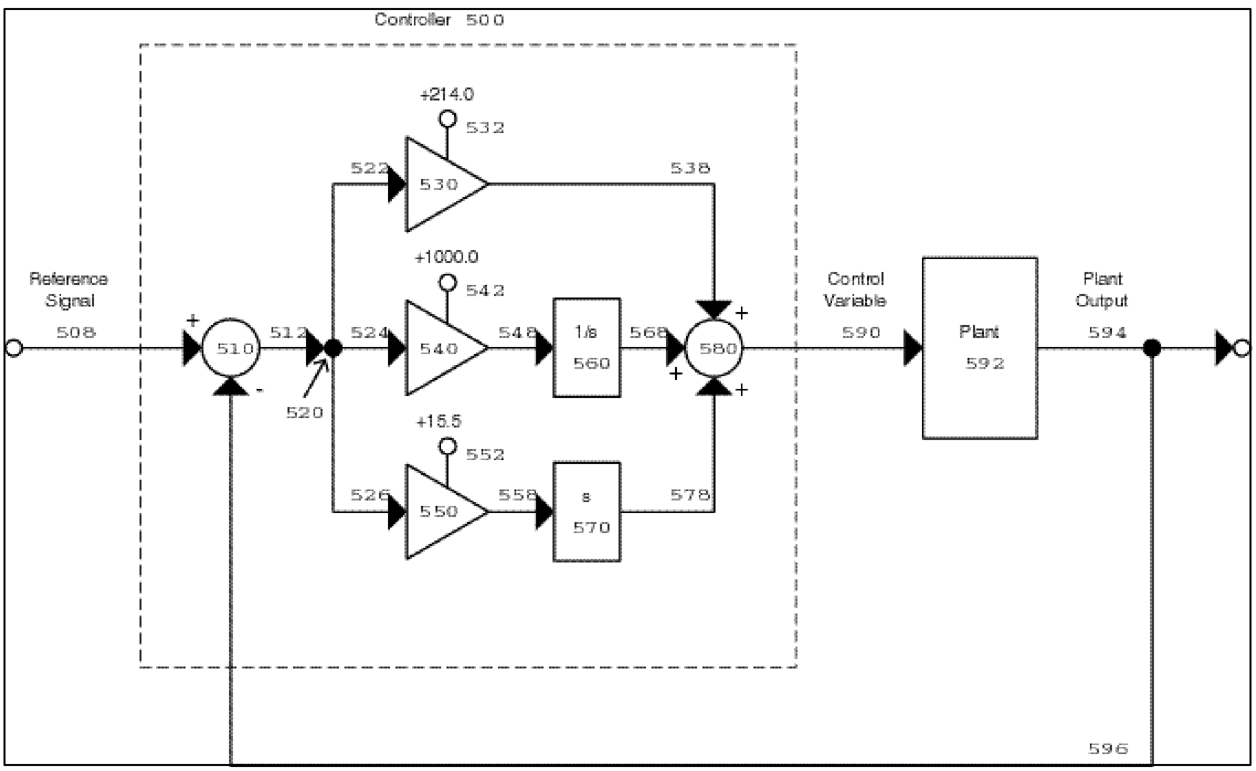
# CONTROLLER BLOCKS

- **gain**
- **integrator**
- **differentiator**
- **adder**
- **subtractor**
- **multiplier**
- **differential input integrators**
- **inverter**
- **lead**
- **lag**
- **two-parameter lag**
- **absolute value**
- **limiter**
- **divider**
- **delay**
- **conditional operators (switches) that operate on time-domain signals**

# SEVEN REPRESENTATIONS FOR A CONTROLLER

- **Block Diagram**
- **Transfer Function involving the Laplace transform variable *s***
- **Program Tree**
- **LISP Symbolic Expression**
- **Mathematica expression (or MATLAB)**
- **Connection List**
- **SPICE Netlist**

# BLOCK DIAGRAM OF A PLANT AND A PID CONTROLLER COMPOSED OF PROPORTIONAL (P), INTEGRATIVE (I), AND DERIVATIVE (D) BLOCKS (WITH EXTERNAL FEEDBACK OF PLANT OUTPUT BACK TO CONTROLLER)



# BLOCK DIAGRAMS FOR CONTROLLERS

**· Controllers are often represented by block diagrams. A block diagram is a graphical structure containing**

**(1) directed lines representing the (one-directional) flow of time-domain signals within the controller,**

**(2) time-domain signal processing blocks (e.g., integrator, differentiator, lead, lag, gain, adder, inverter, and multiplier),**

**(3) external input points from which the controller receives signals (e.g., the reference signal and the plant output that is externally fed back from the plant to the controller), and**

**(4) output point(s) for the controller, conventionally called the *control variable*).**

**· Some signal processing blocks have multiple inputs (e.g., an adder**

# BLOCK DIAGRAMS FOR CONTROLLERS ¾ CONTINUED

· **All signal processing blocks have exactly one output. Because of this restriction, block diagrams for controllers usually contain *takeoff points* that enable the output of a block to be disseminated to more than one other point in the block diagram.**

· **Many blocks used in controllers (e.g., gain, lead, lag, delay) possess numerical parameters. The determination of these values is called *tuning*.**

· **Block diagrams sometimes also contain internal feedback (internal loops in the controller) or feedback of the controller output directly into the controller.**

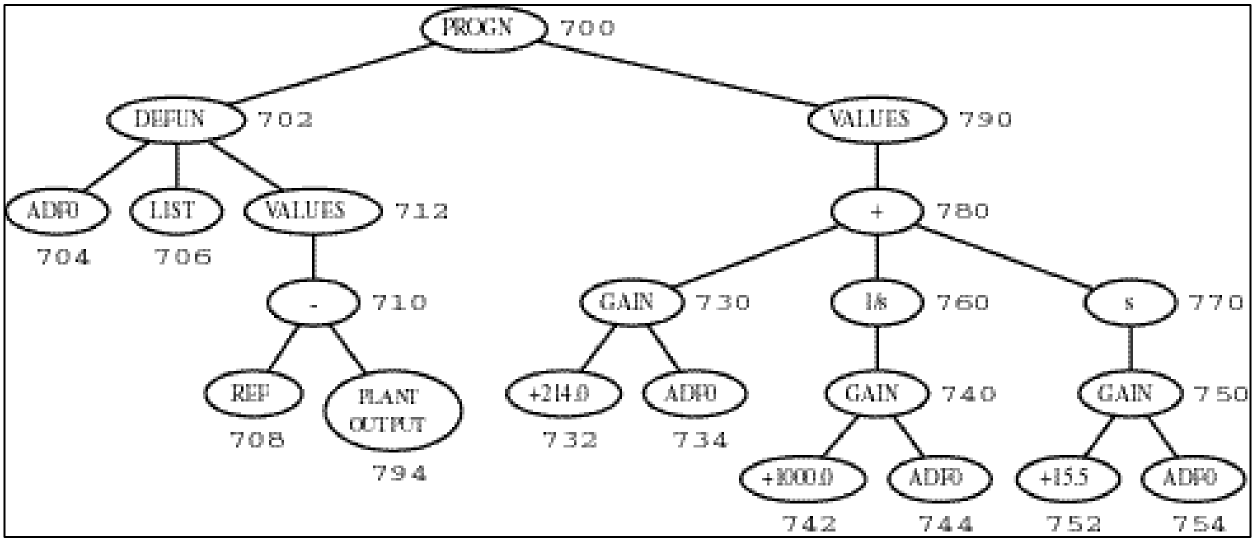
# CONTROLLER SYNTHESIS

· **The design (synthesis) of the design for a controller entails specification of both the topology and parameter values (tuning) for the block diagram of the controller such that the controller satisfies user-specified high-level design requirements. Specifically, the design process for a controller entails making decisions concerning the total number of signal processing blocks to be employed in the controller, the type of each block (e.g., integrator, differentiator, lead, lag, gain, adder, inverter, and multiplier), the interconnections between the inputs and outputs of each signal processing block and between the controller's external input and external output points, and the values of all required numerical parameters for the signal processing blocks.**

# CONTROLLER SYNTHESIS ¾ CONTINUED

**· A program tree can be used to represent the block diagram of a controller. The block diagram consists of time-domain signal processing functions linked by directed lines representing the flow of information. There is no order of evaluation of the functions and terminals of the program tree. Instead, the signal processing blocks of the controller and the plant interact with one another other as part of a closed system in the manner specified by the topology of the block diagram.**

# PROGRAM TREE REPRESENTATION FOR A PID CONTROLLER




```
(PROGN
 (DEFUN ADF0 ()
  (VALUES
   (- REFERENCE_SIGNAL PLANT_OUTPUT)))
 (VALUES
  (+
   (GAIN 214.0 ADF0)
   (DERIVATIVE (GAIN 1000.0 ADF0))
   (INTEGRATOR (GAIN 15.5 ADF0))))
)
```

# AUTOMATICALLY DEFINED FUNCTIONS

· **Automatically defined function `ADF0` can be used as takeoff point**

· **ADF here takes the difference between the reference signal and the plant output and makes this difference available to three points in the result-producing branch**

· **ADFs can also serve as a means for REUSE**

· **There are alternative approaches (e.g., a TAKEOFF function)**

# TRANSFER FUNCTION FOR A PID CONTROLLER INVOLVING THE LAPLACE TRANSFORM VARIABLE *s*

$$G_c(s) = 214.0 + \frac{1000.0}{s} + 15.5s = \frac{214.0s + 1000.0 + 15.5s^2}{s}$$

# PID CONTROLLER

· **The PID controller was patented in 1939 by Albert Callender and Allan Stevenson of Imperial Chemical Limited of Northwich, England.**

· **The PID controller was a significant improvement over earlier and simpler control techniques (which often were merely proportional).**

# PID CONTROLLER

- **Callender and Stevenson (1939) state,**

  **"If the compensating effect *V* is applied in direct proportion to the magnitude of the deviation Q, over-compensation will result. To eliminate the consequent hunting and instability of the system, the compensating effect is additionally regulated in accordance with other characteristics of the deviation in order to bring the system back to the desired balanced condition as rapidly as possible. These characteristics include in particular the rate of deviation (which may be indicated mathematically by the time-derivative of the deviation) and also the summation or quantitative total change of the deviation over a given time (which may be indicated mathematically by the time-integral of the deviation)."**

# PID CONTROLLER

**Callender and Stevenson (1939) also say,**

> **"A specific object of the invention is to provide a system which will produce a compensating effect governed by factors proportional to the total extent of the deviation, the rate of the deviation, and the summation of the deviation during a given period …"**

# PID CONTROLLER

**Claim 1 of Callender and Stevenson (1939) covers what is now called the PI (proportional-integrative) controller,**

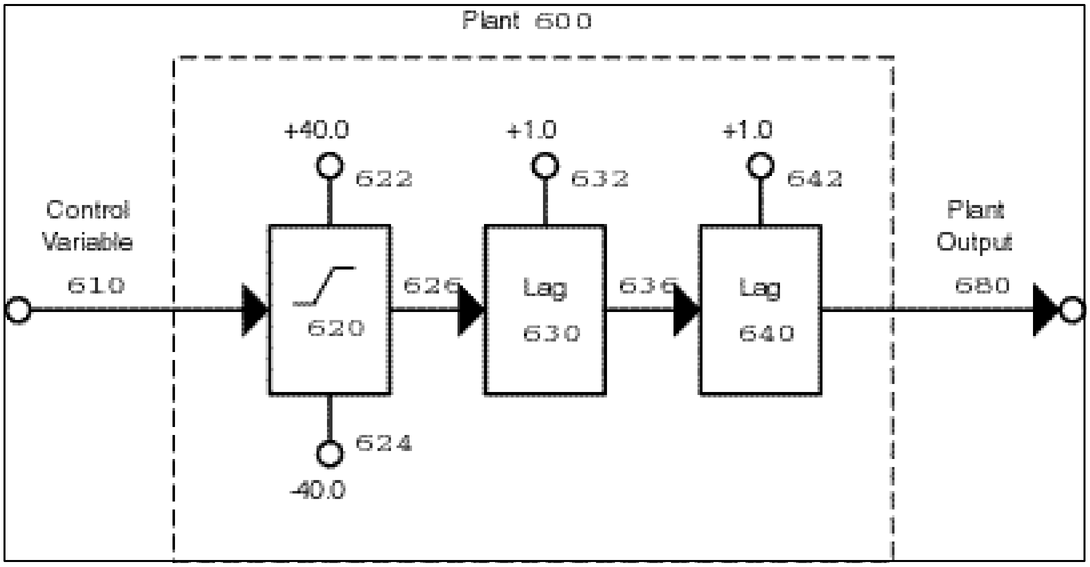
> **"A system for the automatic control of a variable characteristic comprising means proportionally responsive to deviations of the characteristic from a desired value, compensating means for adjusting the value of the characteristic, and electrical means associated with and actuated by responsive variations in said responsive means, for operating the compensating means to correct such deviations in conformity with the sum of the extent of the deviation and the summation of the deviation."**

# PID CONTROLLER

**Claim 3 of Callender and Stevenson (1939) covers what is now called the PID (proportional-integrative-derivative) controller,**

> **"A system as set forth in claim 1 in which said operation is additionally controlled in conformity with the rate of such deviation."**

# TWO-LAG PLANT PROBLEM




· **The two-lag plant has 2 lag blocks, each with a time constant of** t

· **There is a limiter block that constrains the plant's input (the control variable coming from the controller) between -40 and +40 volts before it reaches the lag blocks**

# FUNCTION SET AND TERMINAL SET FOR TWO-LAG PLANT

· **The function set, `F` (for every part of the result-producing branch and any automatically defined functions except the arithmetic-performing subtrees) is**

**`F` = {`GAIN`, `INVERTER`, `LEAD`, `LAG`, `LAG2`, `DIFFERENTIAL_INPUT_INTEGRATOR`, `DIFFERENTIATOR`, `ADD_SIGNAL`, `SUB_SIGNAL`, `ADD_3_SIGNAL`, `ADF0`, `ADF1`, `ADF2`, `ADF3`, `ADF4`}**

· **The terminal set, `T`, (for every part of the result-producing branch and any automatically defined functions except the arithmetic-performing subtrees) is**

**`T` = { `REFERENCE_SIGNAL`, `CONTROLLER_OUTPUT`, `PLANT_OUTPUT`, `CONSTANT_0`}**

# ARITHMETIC-PERFORMING SUBTREES FOR THE TWO-LAG PLANT PROBLEM

· **Signal processing blocks such as GAIN, LEAD, LAG, and LAG2 possess numerical parameter(s)**

· **Parameter values can be established by an arithmetic-performing subtree**

· **A constrained syntactic structure enforces a different function and terminal set for the arithmetic-performing subtrees (as opposed to all other parts of the program tree).**

· **Terminal set, $T_{aps}$, for the arithmetic-performing subtrees**

**$T_{aps}$ = {Â}**

**where Â denotes constant numerical terminals in the range from -1.0 to +1.0**

· **Function set, $F_{aps}$, for the arithmetic-performing subtrees**

**$F_{aps}$ = {ADD_NUMERIC, SUB_NUMERIC}**

# NOTE THAT NOTHING IN THE FUNCTION SET AND TERMINAL SET …

- **mandates use of negative feedback**

- **incorporates any information about the plant**

# FITNESS MEASURE FOR CONTROLLER PROBLEMS

· **The fitness measure is a mathematical implementation of the high-level requirements of the problem and is couched in terms of "what needs to be done" ¾ not "how to do it"**

· **The fitness measure may incorporate any measurable, observable, or calculable behavior or characteristic or combination of behaviors or characteristics**

· **The fitness measure for most problems of controller design is usually multi-objective and involves several different (usually conflicting) requirements**

· **The fitness measure may combine optimization requirements, time-domain constraints, frequency-domain constraints, and robustness requirements into a fitness measure**

# VARIOUS POSSIBLE ELEMENTS OF A FITNESS MEASURE FOR CONTROLLER SYNTHESIS PROBLEMS

- **Optimization metrics**
  - **ITAE ¾ Integral of time-weighted absolute error**
  - **integral of the squared error**
  - **settling time**
  - **rise time**
- **Time-domain constraints**
  - **overshoot**
  - **disturbance rejection**
  - **stability**
- **Frequency-domain constraints**
  - **bandwidth**
  - **AC sweep over the frequencies**
- **Robustness in the face of variation in plant characteristics**
  - **internal gain**
  - **time constant**
- **Robustness in face of added sensor noise**
  - **plant output**
  - **reference signal**
  - **control variable**
  - **plant's internal states (if used)**

# VARIOUS POSSIBLE ELEMENTS TO FITNESS MEASURE FOR CONTROLLER PROBLEMS ¾ CONTINUED

- **Constraints on**
  - **plant's internal states**
  - **control variable**
- **Robustness of plant's behavior to changes in some external variable**
  - **temperature**
  - **plant's flow rate**
- **Consistency in the face of variations in the step size of the reference signal**

- **Intermixing of different types of considerations is difficult (sometimes impossible) when conventional analytical techniques are used to design controllers**

# FITNESS MEASURE FOR CONTROLLER PROBLEMS ¾ CONTINUED

· **The program tree (i.e., the result-producing branch and any automatically defined functions) is executed to produce a block diagram for the controller**

· **The netlist for the resulting controlled system (i.e., the controller and the to-be-controlled plant) is determined**

· **The controller is then simulated using our modified version of the SPICE simulator**

· **The netlist is wrapped inside an appropriate set of simulator commands**

· **The simulator returns tabular information**

· **Non-simulatable controller is assigned a high penalty value of fitness ($10^8$)**

· **If controller takes more than a specified amount of computer time, the simulation is terminated and fitness becomes $10^8$**

# FITNESS MEASURE FOR TWO-LAG PLANT

- **Fitness is sum (i.e., linear combination) of the detrimental contributions of 10 elements**
- **The smaller the sum, the better**
  - **8 time-domain-based elements based on a modified integral of time-weighted absolute error measuring the achievement of the desired value of the plant response, the controller's robustness, and the controller's avoidance of overshoot**
  - **1 time-domain-based element measuring the controller's stability when faced with an extreme spiked reference signal**
  - **1 frequency-domain-based element measuring the reasonableness of the controller's frequency response**

# FITNESS MEASURE FOR TWO-LAG PLANT ¾ CONTINUED

· **The first eight elements of the fitness measure represent the eight choices of a particular one of two different values of the plant's internal gain, *K*, in conjunction with a particular one of two different values of the plant's time constant $t$, in conjunction with a particular one of two different values for the height of the reference signal.**

- **The two values of *K* are 1.0 and 2.0.**
- **The two values of $t$ are 0.5 and 1.0.**
- **The two reference signals are step functions that rise from 0 to 1 volts (or 1 microvolts) at *t* = 100 milliseconds.**

# FITNESS MEASURE FOR TWO-LAG PLANT ¾ CONTINUED

**· For each of these eight fitness cases, a transient analysis is performed in the time domain using the SPICE simulator. The contribution to fitness for each of these eight elements of the fitness measure is based on the integral of time-weighted absolute error**

$$\int_{t=0}^{9.6} t\left|e(t)\right|A(e(t))B\,dt$$

**·** ***e*****(*****t*****) is the difference (error) at time** ***t*** **between the plant output and the reference signal.**

**· The multiplication of each value of** ***e*****(*****t*****) by** ***B*** **($10^6$. or 1) makes both reference signals equally influential.**

**· The additional weighting function,** ***A*****, heavily penalizes non-compliant amounts of overshoot.** ***A*** **weights all variations up to 2% above the reference signal by 1.0, and all overshoots above 2% by 10.0.**

# FITNESS MEASURE FOR TWO-LAG PLANT ¾ CONTINUED

· **The 9th element of the fitness measure exposes the controller to an extreme spiked reference signal.**

· **The spiked reference signal rises to $10^{-9}$ volts at time *t* = 0 and persists for 10-nanoseconds. A transient analysis is performed using the SPICE simulator for 121 fitness cases representing times *t* = 0 to *t* = 120 microseconds. If the plant output never exceeds a fixed limit of $10^{-8}$ volts (i.e., a order of magnitude greater than the pulse's magnitude), then this element of the fitness measure is zero. However, if the absolute value of plant output goes above $10^{-8}$ volts for any time *t*, then the contribution to fitness is 500(0.000120 - *t*). This penalty is a ramp starting at the point (0, 0.06) and ending at the point (1.2, 0), so that 0.06 seconds is the maximum penalty.**

# FITNESS MEASURE FOR TWO-LAG PLANT ¾ CONTINUED

- **The 10th element constrains the frequency of the control variable so as to avoid extreme high frequencies. If the closed loop frequency response is acceptable, this element of the fitness measure will be zero. SPICE is instructed to perform an AC sweep over 121 frequencies over six decades of frequency between 0.01 Hz and 10,000 Hz. A gain of 0 dB is ideal for the 80 fitness cases in the first four decades of frequency; however, a gain of up to +3 dB is acceptable. The contribution is zero if the gain is ideal or acceptable, but 18/121 per fitness case otherwise. The maximum acceptable gain for the 41 fitness cases in the last two decades is given by the straight line connecting (100 Hz, -3 dB) and (10,000 Hz, -83 dB). The contribution is zero if the gain is below this straight line, but otherwise 18/121 per fitness case.**

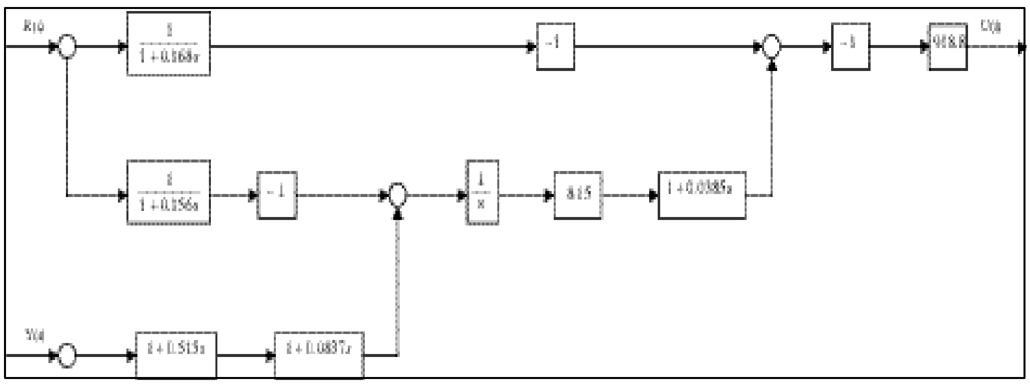
# BEST OF GENERATION 0 FOR THE TWO-LAG PLANT

· **The best individual from generation 0 has a fitness of 8.26. The S-expression is shown below (except that, for simplicity, the 29-point arithmetic-performing subtree establishing the amplification factor for the `GAIN` function has been replaced by the equivalent numerical value of 62.8637)**

```
(gain
 (differentiator

(differential_input_integrator
   (lag reference_signal
0.708707)
   plant_output
  )
 )
62.8637)
```

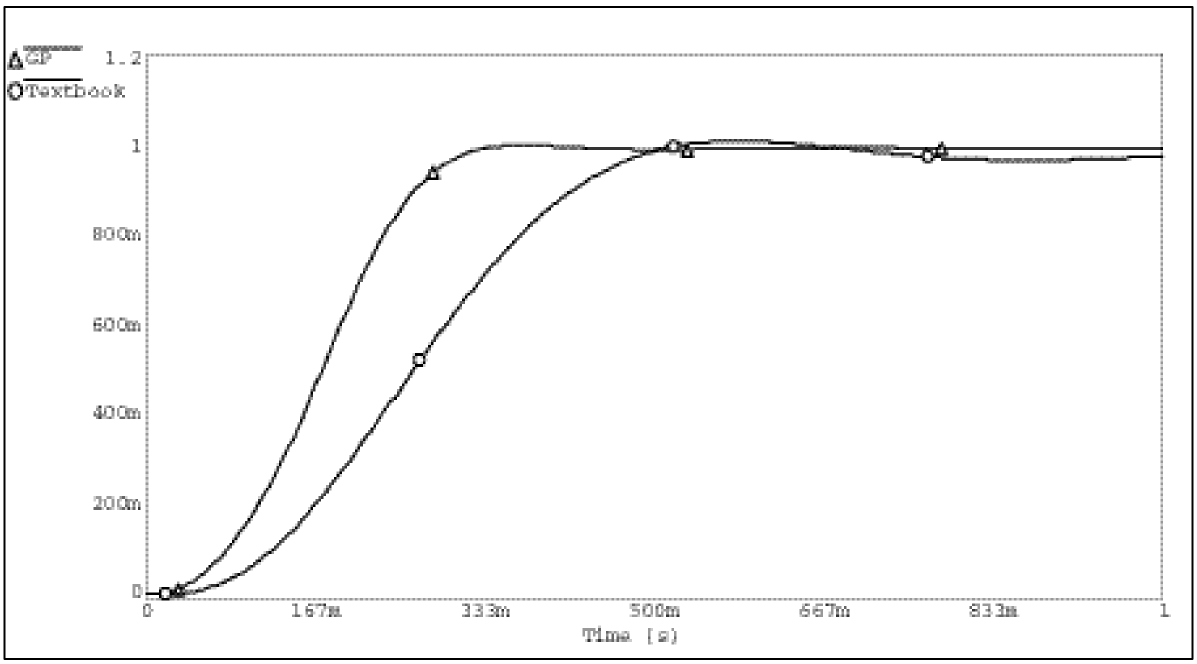
# BEST-OF-RUN GENETICALLY EVOLVED CONTROLLER FROM GENERATION 32 FOR THE TWO-LAG PLANT



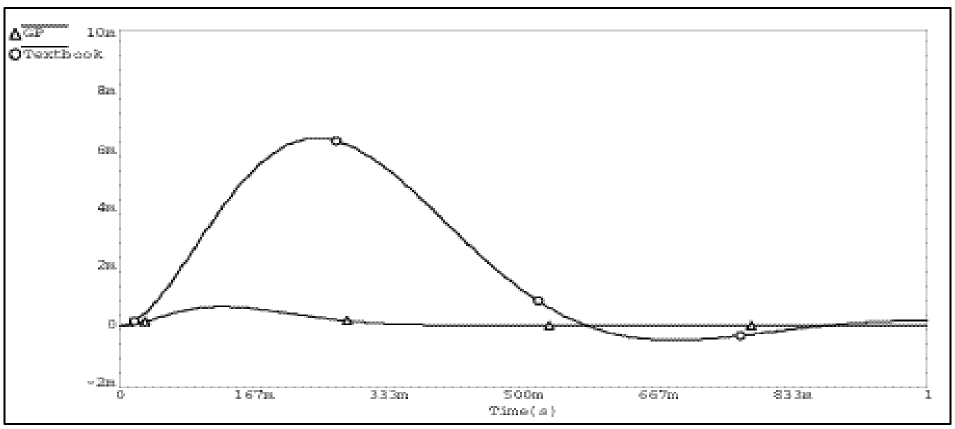
# FITNESS OF BEST-OF-RUN INDIVIDUAL OF GENERATION 32 FOR THE TWO-LAG PLANT PROBLEM

| | Step size (volts) | Internal Gain, *K* | Time constant, *t* | Fitness |
|---|---|---|---|---|
| 0 | 1 | 1 | 1.0 | 0.0220 |
| 1 | 1 | 1 | 0.5 | 0.0205 |
| 2 | 1 | 2 | 1.0 | 0.0201 |
| 3 | 1 | 2 | 0.5 | 0.0206 |
| 4 | $10^{-6}$ | 1 | 1.0 | 0.0196 |
| 5 | $10^{-6}$ | 1 | 0.5 | 0.0204 |
| 6 | $10^{-6}$ | 2 | 1.0 | 0.0210 |
| 7 | $10^{-6}$ | 2 | 0.5 | 0.0206 |
| 8 | Spiked reference signal | | | 0.0000 |
| 9 | AC sweep | | | 0.0000 |
| TOTAL FITNESS | | | | 0.1639 |

# COMPARISON OF THE TIME-DOMAIN RESPONSE TO 1-VOLT STEP INPUT FOR THE EVOLVED CONTROLLER (TRIANGLES) AND THE BISHOP AND DORF CONTROLLER (SQUARES) FOR THE TWO-LAG PLANT WITH *K*=1 AND t=1



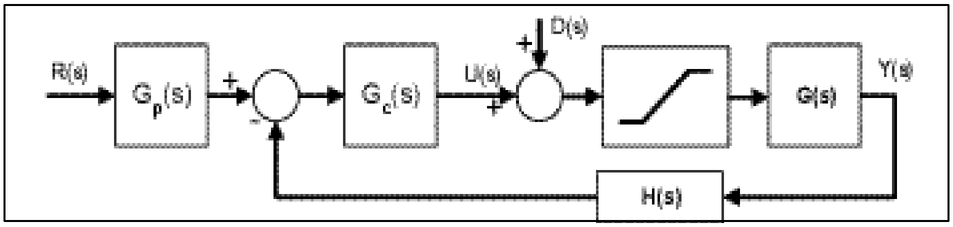
# COMPARISON OF THE TIME-DOMAIN RESPONSE TO A 1-VOLT DISTURBANCE SIGNAL OF THE EVOLVED CONTROLLER(TRIANGLES) AND THE BISHOP AND DORF CONTROLLER (CIRCLES) FOR THE TWO-LAG PLANT WITH *K*=1 AND t=1



# COMPARISON FOR THE TWO-LAG PLANT

| | Units | Genetically evolved controller | Dorf and Bishop (Dorf and Bishop 1998) |
|---|---|---|---|
| **Disturbance sensitivity** | μVolts /Volt | 644 | 5,775 |
| **ITAE** | millivolt $sec^2$ | 19 | 46 |
| **Bandwidth (3 dB)** | Hz | 1.5 | 1 |
| **Rise time** | milliseconds | 296 | 465 |
| **Settling time** | milliseconds | 304 | 944 |

# OVERALL MODEL




· **The reference signal $R(s)$ is fed through pre-filter $G_p(s)$. The plant output $Y(s)$ is passed through $H(s)$ and then subtracted, in continuous time, from the pre-filtered reference signal and the difference (error) is fed into the compensator $G_c(s)$. The plant $G(s)$ has one input and one output $Y(s)$. $G_c(s)$ has one input (the difference) and one output $U(s)$. Disturbance $D(s)$ may be added to the output $U(s)$ of $G_c(s)$. The resulting sum is subjected to a limiter (in the range between -40 and +40 volts for this problem).**

# RESULTS ¾ TWO-LAG PLANT

· **The transfer function for the pre-filter, $G_{p\text{-}dorf}(s)$, of the Dorf and Bishop controller is**

$$G_{p-dorf}(s) = \frac{42.67}{42.67 + 11.38s + s^2}$$

**and the transfer function for their PID compensator, $G_{c\text{-}dorf}(s)$, is**

$$G_{c-dorf}(s) = \frac{12(42.67 + 11.38s + s^2)}{s}.$$

· **After applying standard block diagram manipulations, the transfer function for the best-of-run controller from generation 32 for the two-lag plant can be expressed as a transfer function for a pre-filter and a transfer function for a compensator. The transfer function for the pre-filter, $G_{p32}(s)$, for the best-of-run individual from generation 32 for the two-lag plant is**

$$G_{p32}(s) = \frac{1(1 + .1262s)(1 + .2029s)}{(1 + .03851s)(1 + .05146s)(1 + .08375)(1 + .1561s)(1 + .1680s)}$$

**The transfer function for the compensator, $G_{c32}(s)$, for the best-of-run individual from generation 32 for the two-lag plant is**

$$G_{c32}(s) = \frac{7487(1 + .03851s)(1 + .05146s)(1 + .08375s)}{s} = \frac{7487.05 + 1300.63s + 71.2511s^2 + 1.2426s^3}{s}$$

# RESULTS ¾ TWO-LAG PLANT

**· The $s^3$ term (in conjunction with the *s* in the denominator) indicates a second derivative. Although derivatives may not be useful in some controllers (because they may amplify high frequency effects such as noise), their use here is appropriate since there are no such possibly disadvantageous effects in this particular problem. Thus, the evolved compensator is a PID-D2 (proportional, integrative, derivative, and second derivative) controller.**

**· Dorf and Bishop solved this problem by seeking the parameter values (tuning) for a PID-type controller. However, genetic programming was not encumbered by the kind of preconceptions that often channel human thinking along well-trodden paths. Instead, genetic programming starts each run as a new adventure that is free to innovate in any manner that may satisfy the requirements of the problem.**

# JONES 1942 PATENT

**Harry Jones of The Brown Instrument Company of Philadelphia patented the PID-D2 controller topology in 1942. As Jones states,**

> **"A … specific object of the invention is to provide electrical control apparatus … wherein the rate of application of the controlling medium may be effected in accordance with or in response to the first, second, and high derivatives of the magnitude of the condition with respect to time, as desired."**

# JONES 1942 PATENT

**Claim 38 of the Jones 1942 patent (Jones 1942) states,**

> **"In a control system, an electrical network, means to adjust said network in response to changes in a variable condition to be controlled, control means responsive to network adjustments to control said condition, reset means including a reactance in said network adapted following an adjustment of said network by said first means to initiate an additional network adjustment in the same sense, and rate control means included in said network adapted to control the effect of the first mentioned adjustment in accordance with the second or higher derivative of the magnitude of the condition with respect to time."**

# JONES 1942 PATENT

· **Because the best-of-run individual from generation 32 has proportional, integrative, derivative, and second derivative blocks, it infringes on the 1942 Jones patent.**

· **The legal criteria for obtaining a U. S. patent are that the proposed invention be "new" and "useful" and that**

> **" ¼ the differences between the subject matter sought to be patented and the prior art are such that the subject matter as a whole would (not) have been obvious at the time the invention was made to a person having ordinary skill in the art to which said subject matter pertains." (35 *United States Code* 103a)**

# THREE-LAG PLANT PROBLEM

· **The three-lag plant has 3 lag blocks with a time constant of t. The transfer function of the three-lag plant is**

$$G(s) = \frac{K}{(1+ts)^3}$$

· **The plant's internal gain, *K*, is varied from 1 to 2 and the plant's time constant, t, is varied from 0.5 to 1.0.**

· **The control variable is limited to the range between -10 and +10 volts**

# PERTURBABLE NUMERICAL VALUES ARE USED TO SET NUMERICAL PARAMETER VALUES FOR THE THREE-LAG PLANT PROBLEM

- **Parameter values are established by a perturbable numerical value**
- **A constrained syntactic structure permits only a single perturbable numerical value to appear as the argument for establishing each numerical parameter value for each signal processing block taking a numerical parameter**

- **Terminal set, $T_{aps}$, for the arithmetic-performing subtrees**

**$T_{aps}$ = {Â}**

**where Â denotes constant numerical terminals in the range from -1.0 to +1.0**

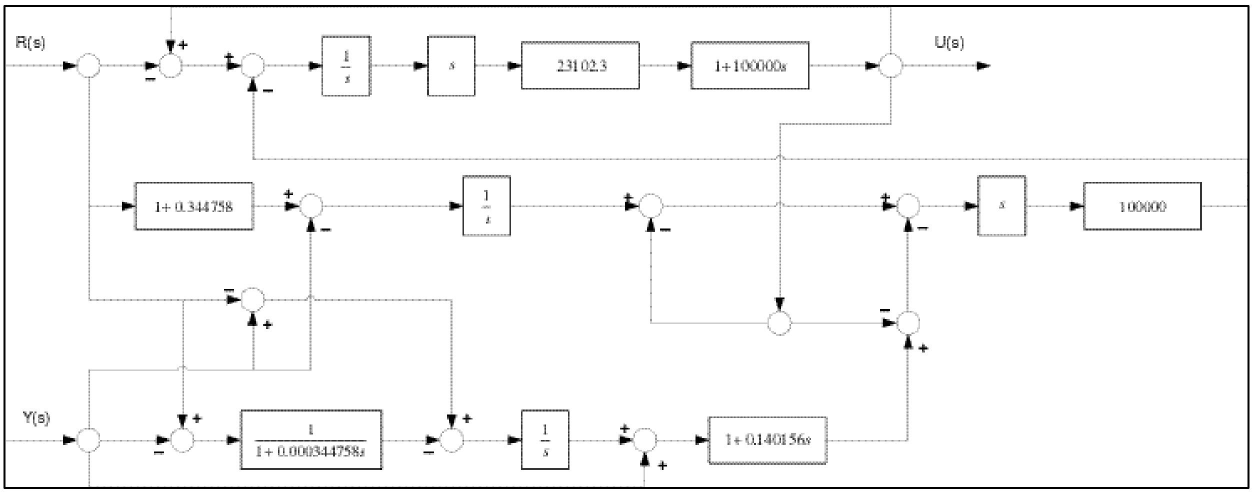
# FITNESS MEASURE FOR THE THREE-LAG PLANT PROBLEM

· **For the three-lag plant problem, the fitness of a controller is measured using 10 elements.**

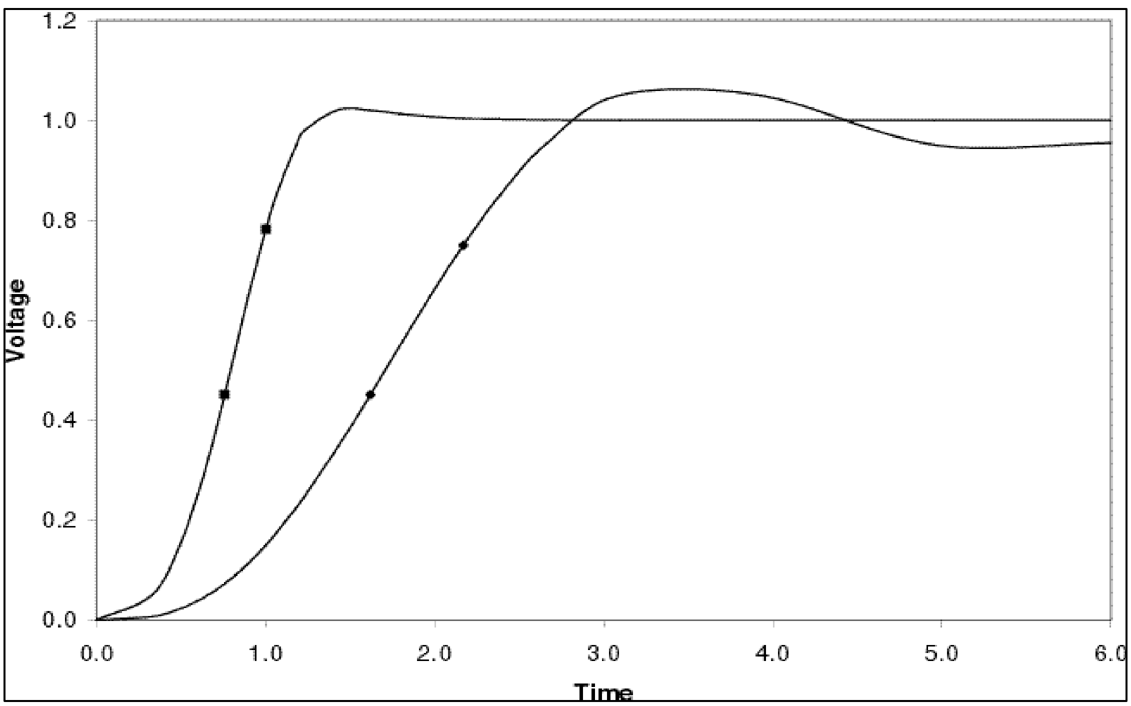
· **The first nine elements are the same as for the two-lag plant problem.**

· **The $10^{th}$ element is based on disturbance rejection. The reference signal is held at a 0. A disturbance signal consisting of a unit step is added to the controller variable (plant input) at time $t$ = 0. The resulting disturbed signal is provided as input to the plant. The detrimental contribution to fitness is the absolute value of the largest single difference between the plant output and the reference signal (which is invariant at 0 throughout).**

# BEST-OF-RUN GENETICALLY EVOLVED CONTROLLER FROM GENERATION 31 FOR THE THREE-LAG PLANT



# COMPARISON OF THE TIME-DOMAIN RESPONSE TO A 1 VOLT UNIT STEP OF THE BEST-OF-RUN EVOLVED CONTROLLER (SQUARES) FROM GENERATION 31 AND THE ASTROM AND HAGGLUND CONTROLLER (CIRCLES) FOR THE THREE-LAG PLANT WITH *K*=1 AND τ=1



# CHARACTERISTICS OF BEST-OF-RUN INDIVIDUAL OF GENERATION 31 FOR THE THREE-LAG PLANT

| Step size | *K* | *t* | Disturbance | ITAE | Bandwidth | Rise time | Settling time |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1.0 | 4.3 | 0.360 | 0.72 | 1.25 | 1.87 |
| 1 | 1 | 0.5 | 4.3 | 0.190 | 0.72 | 0.97 | 1.50 |
| 1 | 2 | 1.0 | 4.3 | 0.240 | 0.72 | 0.98 | 1.39 |
| 1 | 2 | 0.5 | 4.3 | 0.160 | 0.72 | 0.90 | 1.44 |
| $10^{-6}$ | 1 | 1.0 | 4.3 | 0.069 | 0.72 | 0.64 | 1.15 |
| $10^{-6}$ | 1 | 0.5 | 4.3 | 0.046 | 0.72 | 0.53 | 0.97 |
| $10^{-6}$ | 2 | 1.0 | 4.3 | 0.024 | 0.72 | 0.34 | 0.52 |
| $10^{-6}$ | 2 | 0.5 | 4.3 | 0.046 | 0.72 | 0.52 | 0.98 |
| | | AVERAGE | 4.3 | 0.142 | 0.72 | 0.77 | 1.23 |

# CHARACTERISTICS OF PID SOLUTION (ASTROM AND HAGGLUND 1995) FOR THE THREE-LAG PLANT

| | Step size | *K* | *t* | Disturbance | ITAE | Bandwidth | Rise time | Settling time |
|---|---|---|---|---|---|---|---|---|
| **1** | **1** | **1** | **1.0** | **186,000** | **2.6** | **0.248** | **2.49** | **6.46** |
| **2** | **1** | **1** | **0.5** | **156,000** | **2.3** | **0.112** | **3.46** | **5.36** |
| **3** | **1** | **2** | **1.0** | **217,000** | **2.0** | **0.341** | **2.06** | **5.64** |
| **4** | **1** | **2** | **0.5** | **164,000** | **1.9** | **0.123** | **3.17** | **4.53** |
| **5** | **$10^{-6}$** | **1** | **1.0** | **186,000** | **2.6** | **0.248** | **2.49** | **6.46** |
| **6** | **$10^{-6}$** | **1** | **0.5** | **156,000** | **2.3** | **0.112** | **3.46** | **5.36** |
| **7** | **$10^{-6}$** | **2** | **1.0** | **217,000** | **2.0** | **0.341** | **2.06** | **5.64** |
| **8** | **$10^{-6}$** | **2** | **0.5** | **164,000** | **1.9** | **0.123** | **3.17** | **4.53** |
| | | | AVERAGE | **180,750** | **2.2** | **0.21** | **2.8** | **5.5** |

# COMPARISON OF AVERAGE CHARACTERISTICS FOR THE THREE-LAG PLANT

| | **Units** | **Genetically evolved controller** | **PID controller** |
|---|---|---|---|
| **Disturbance sensitivity** | **mvolts /volt** | **4.3** | **180,750** |
| **ITAE** | **millivolt seconds$^2$** | **0.142** | **2.2** |
| **Bandwidth (3 dB)** | **Hertz** | **0.72** | **0.21** |
| **Rise time** | **milliseconds** | **0.77** | **2.8** |
| **Settling time** | **milliseconds** | **1.23** | **5.5** |

# PROBLEM OF THREE-LAG PLANT WITH A SIGNIFICANT (FIVE-SECOND) TIME DELAY

- **The illustrative problem entails creation of both the topology and parameter values for a controller for a three-lag plant with a significant (five-second) time delay in the external feedback from the plant output to the controller such that plant output reaches the level of the reference signal in minimal time (as measured by the integral of the time-weighted absolute error), such that the overshoot in response to a step input is less than 2%, and such that the controller is robust in the face of disturbance (added into the controller output). The delay in the feedback makes the design of an effective controller especially difficult (Astrom and Hagglund 1995). The transfer function of the plant is**

$$G(s) = \frac{Ke^{-5s}}{(1+ts)^3}$$

# PROBLEM OF THREE-LAG PLANT WITH A SIGNIFICANT (FIVE-SECOND) TIME DELAY ¾ CONTINUED

· **To make the problem more realistic, we added an additional constraint (satisfied by the controller presented in Astrom and Hagglund 1995) that the input to the plant is limited to the range between -40 and +40 volts. The plant in this paper operates over several different combinations of values for *K* and** t **(whereas the controller designed by Astrom and Hagglund was intended only for *K* = 1 and** t **= 1).**

# PREPARATORY STEPS

## PROGRAM ARCHITECTURE

**· Since there is one result-producing branch in the program tree for each output from the controller and this problem involves a one-output controller, each program tree has one result-producing branch. Each program tree in the initial random generation (generation 0) has no automatically defined functions. However, in subsequent generations, architecture-altering operations may insert and delete automatically defined functions (up to a maximum of five per program tree).**

## TERMINAL SET

**· A constrained syntactic structure permits only a single perturbable numerical value to appear as the argument for establishing each numerical parameter value for each signal processing block requiring a parameter value. These numerical values initially range**

**from -5.0 to +5.0. These numerical values are perturbed during the run by a Gaussian mutation operation that operates only on numerical values. Numerical constants are later interpreted on a logarithmic scale so that they represent values in a range of 10 orders of magnitude (Koza, Bennett, Andre, and Keane 1999).**

**· The remaining terminals are time-domain signals. The terminal set, T, for the result-producing branch and any automatically defined functions (except for the perturbable numerical values mentioned above) is**

**T = {REFERENCE_SIGNAL, CONTROLLER_OUTPUT, PLANT_OUTPUT, CONSTANT_0}**

# FUNCTION SET

· **The functions are signal processing functions that operate on time-domain signals (the terminals in T). The function set, F, for the result-producing branch and any automatically defined functions is**

**F = {GAIN, INVERTER, LEAD, LAG, LAG2, DIFFERENTIAL_INPUT_INTEGRATOR, DIFFERENTIATOR, ADD_SIGNAL, SUB_SIGNAL, ADD_3_SIGNAL, DELAY, ADF0, ..., ADF4}**

# FITNESS

**· The fitness of a controller is measured using 13 elements consisting of 12 time-domain-based elements based on a modified integral of time-weighted absolute error (ITAE) and one time-domain-based element measuring disturbance rejection.**

**· The fitness of an individual controller is the sum of the detrimental contributions of these 13 elements of the fitness measure.**

# FITNESS ¾ CONTINUED

**· The first 12 elements of the fitness measure evaluate how quickly the controller causes the plant to reach the reference signal and the controller's success in avoiding overshoot. Two reference signals are used. The first reference signal is a step function that rises from 0 to 1 volts at *t* = 100 milliseconds while the second rises from 0 to 1 microvolts at *t* = 100 milliseconds. Two values of the time constant, t, are used (namely 0.5 and 1.0). Three values of *K* are used, namely 0.9, 1.0, and 1.1. Exposing genetic programming to different combinations of values of step size, *K*, and t produces a robust controllers and also prevents genetic programming from engaging in pole elimination.**

**· The contribution of each of these 12 elements is based on ITAE**

$$\int_{t=5}^{36} (t-5)\left|e(t)\right| A(e(t)) BC dt$$

# FITNESS - CONTINUED

· **Because of the built-in five-second time delay, the integration runs from time *t* = 5 seconds to *t* = 36 seconds.**

· **Here *e*(*t*) is the difference (error) at time *t* between the delayed plant output and the reference signal. The integral of time-weighted absolute error penalizes differences that occur later more heavily than differences that occur earlier.**

· **We used a discrete approximation to the integral by considering 120 300-millisecond time steps between *t* = 5 to *t* = 36 seconds.**

# FITNESS - CONTINUED

· **We multiplied each fitness case by the reciprocal of the amplitude of the reference signals so that both reference signals (1 microvolt and 1 volt) are equally influential. Specifically, *B* is a factor that is used to normalize the contributions associated with the two step functions. *B* multiplies the difference *e*(*t*) associated with the 1-volt step function by 1 and multiplies the difference *e*(*t*) associated with the 1-microvolt step function by $10^6$.**

· **The integral contains an additional weight, *A*, that varies with *e*(*t*). The function *A* weights all variation up to 102% of the reference signal by a factor of 1.0, and heavily penalizes overshoots over 2% by a factor 10.0.**

· **The integral contains a special weight, *C*, which is 5.0 for the two fitness cases for which *K* = 1 and t = 1, and 1.0 otherwise.**

# FITNESS - CONTINUED

· **The 13th element of the fitness measure is based on disturbance rejection. The penalty is computed based on a time-domain analysis for 36.0 seconds. The reference signal is held at a value of 0. A disturbance signal consisting of a unit step is added to the `CONTROLLER_OUTPUT` at time *t* = 0. The detrimental contribution to fitness is 500/36 times the time required to bring the plant output to within 20 millivolts of the reference signal of 0 volts (i.e., to reduce the effect to within 2% of the 1-volt disturbance signal) assuming that the plant settles to within this range within 36 seconds. If the plant does not settle to within this range within 36 seconds, the detrimental contribution to fitness is 500 plus the absolute value of the plant output in volts times 500. For example, if the effect of the disturbance was never reduced below 1 volts, the detrimental contribution to fitness would be 1000.**

# CONTROL PARAMETERS

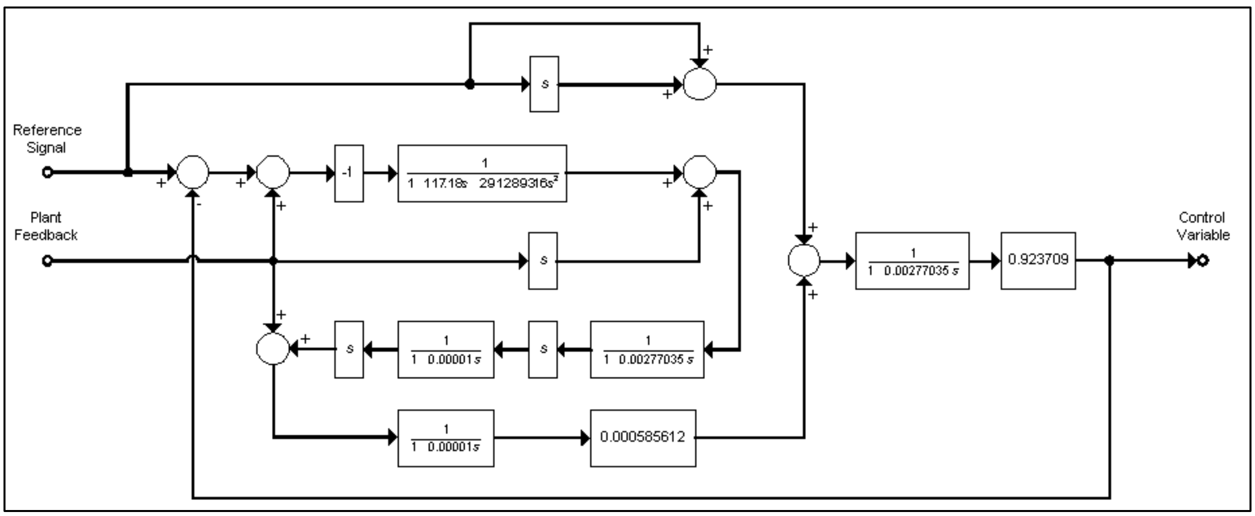
· **The population size, *M*, was 500,000.**

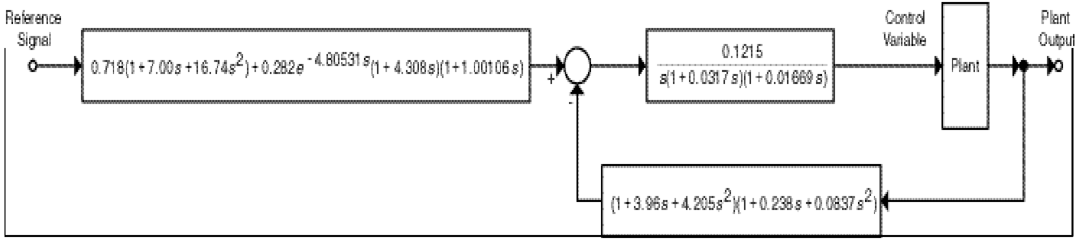
· **A maximum size of 150 points was established for the RPB and 100 points was established for each ADF**

# TERMINATION

**· THE RUN WAS MANUALLY MONITORED AND MANUALLY TERMINATED WHEN THE FITNESS OF MANY SUCCESSIVE BEST-OF-GENERATION INDIVIDUALS APPEARED TO HAVE REACHED A PLATEAU.**

# EXAMPLE OF FEEDBACK OF CONTROLLER OUTPUT INTO THE CONTROLLER CONTAINED IN BEST-OF-GENERATION INDIVIDUAL FROM GENERATION 82 FOR PLANT WITH FIVE-SECOND DELAY



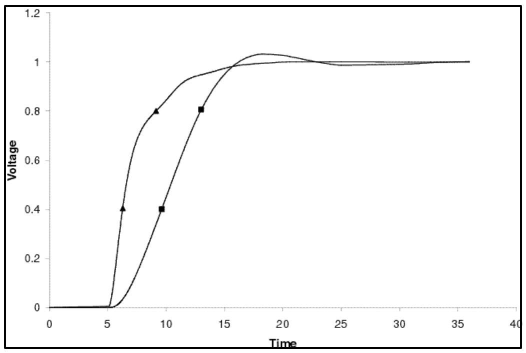


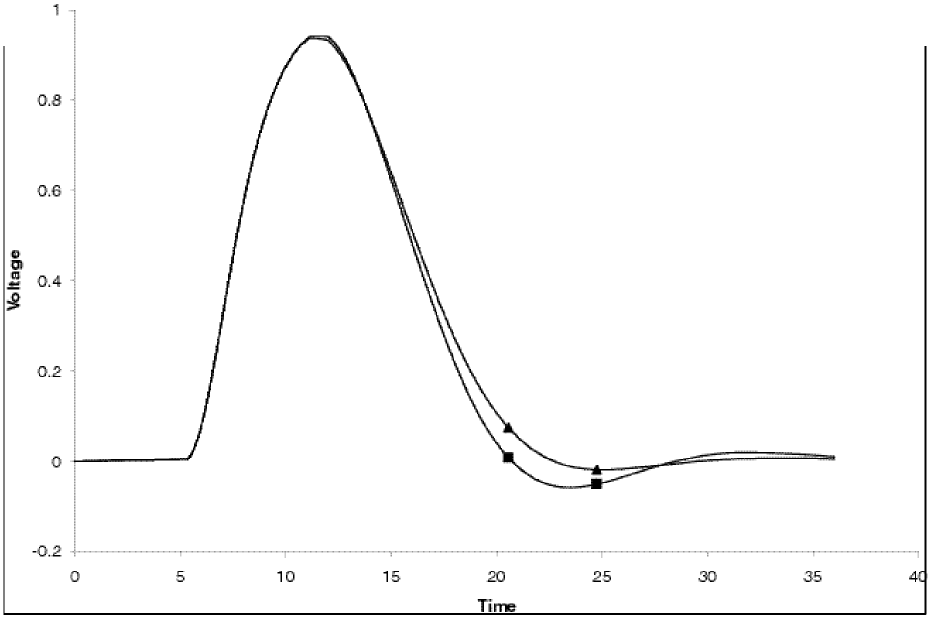

# BEST-OF-RUN CONTROLLER FROM GENERATION 129 FOR THREE-LAG PLANT WITH (SUBSTANTIAL) FIVE-SECOND DELAY

# FITNESS OF BEST-OF-RUN CONTROLLER EMERGED IN GENERATION 129 AND ASTROM AND HAGGLUND CONTROLLER FOR THREE-LAG PLANT WITH FIVE-SECOND DELAY

| | Step size (volts) | Plant internal Gain, *K* | Time constant, *t* | Best-of-run generation 129 | Astrom and Hagglund controller |
|---|---|---|---|---|---|
| 0 | 1 | 0.9 | 1.0 | 13.7 | 27.4 |
| 1 | 1 | 0.9 | 0.5 | 25.6 | 38.2 |
| 2 | 1 | 1.0 | 1.0 | 34.0 / 5 = 6.8 | 22.9 |
| 3 | 1 | 1.0 | 0.5 | 18.6 | 29.3 |
| 4 | 1 | 1.1 | 1.0 | 4.4 | 25.4 |
| 5 | 1 | 1.1 | 0.5 | 16.3 | 22.7 |
| 6 | $10^{-6}$ | 0.9 | 1.0 | 13.2 | 27.4 |
| 7 | $10^{-6}$ | 0.9 | 0.5 | 25.5 | 38.2 |
| 8 | $10^{-6}$ | 1.0 | 1.0 | 30.7 / 5 = 6.1 | 22.9 |
| 9 | $10^{-6}$ | 1.0 | 0.5 | 18.5 | 29.3 |
| 10 | $10^{-6}$ | 1.1 | 1.0 | 4.3 | 25.4 |
| 11 | $10^{-6}$ | 1.1 | 0.5 | 16.2 | 22.7 |
| Disturbance | 1 | 1 | 1 | 302 | 373 |

# COMPARISON FOR STEP INPUT FOR BEST-OF-RUN CONTROLLER FROM GENERATION 129 FOR THREE-LAG PLANT WITH (SUBSTANTIAL) FIVE-SECOND DELAY






**COMPARISON FOR DISTURBANCE REJECTION FOR STEP INPUT OF BEST-OF-RUN CONTROLLER EMERGED IN GENERATION 129 AND ASTROM AND HAGGLUND CONTROLLER**