AUTOMATIC SYNTHESIS OF PARAMETERIZED TOPOLOGIES FOR CONTROLLERS

THREE-LAG PLANT

TWO FAMILIES OF PLANTS

FREE VARIABLES AND CONDITIONAL OPERATORS

• One of the most important characteristics of computer programs is that they ordinarily contain inputs (free variables) and conditional operations.

• Genetic algorithms and other techniques of genetic and evolutionary computation are typically used to search for an optimal (or near-optimal) solution to a particular single instance of a problem.

• EXAMPLE: Find the numerical values for the real and imaginary parts of the two complex roots of a particular quadratic equation, such as $3x^2 + 4x + 5$, a separate run is required to solve each different instance of the problem (e.g., $10x^2 + 2x + 1$).

FREE VARIABLES AND CONDITIONAL OPERATORS

- **Free variables enable a single program to produce different outputs based on the particular values of its free variables.**

- **Conditional operations enable a single program to execute alternative sequences of steps based on the particular values of the free variables.**

- **Conditional operations and free variables together potentially enable a single program to solve all instances of a problem (instead of just one instance of the problem). Thus, a computer program containing free variables and conditional operations has the ability to solve all quadratic equations — $ax^2 + bx + c$**

CONTROLLER WITH A FREE VARIABLE

• **The problem is to create both the topology and parameter values for a robust "generalized" controller for a three-lag plant, where the controller contains a free variable representing the plant's time constant, τ. This free variable, τ, is allowed to range over two orders of magnitude.**

• **The transfer function of the three-lag plant is**

FREE VARIABLE — CONTINUED

- **The controller is to be designed so that plant output reaches the level of the reference signal so as to minimize the integral of the time-weighted error (ITAE), such that the overshoot in response to a step input on the reference signal (setpoint) is less than 2%, and such that the controller is robust in the face of variation in the plant's internal gain, *K* (tested by values of 1.0 and 2.0).**

- **The input to the plant is limited to the range between -40 and +40 volts.**

FREE VARIABLE — CONTINUED

- **Program Architecture: Since the problem involves a one-output controller, each program tree in the population has one result-producing branch.**
- **Terminal Set: An arithmetic-performing subtree containing perturbable numerical terminals, $\Re$, (ranging from -3.0 and +3.0), arithmetic operations, and a free variable consisting of the plant's time constant, τ, is used to establish the numerical parameter value for each signal processing block possessing a parameter.**
  - **The terminal set, T_{aps}, for the arithmetic-performing subtrees is**

    $T_{aps} = \{\Re, \tau\}$
  - **The remaining terminals are time-domain signals. The terminal set, T, for the result-producing branch and any automatically defined functions (except the arithmetic-performing subtrees) is**

    T = {REFERENCE_SIGNAL, CONTROLLER_OUTPUT, PLANT_OUTPUT, CONSTANT_0}.

FREE VARIABLE — CONTINUED

- **Function Set: The function set, F_{aps}, for the arithmetic-performing subtrees consists of the following functions:**

F_{aps} = {ADD_NUMERIC, SUB_NUMERIC, MUL_NUMERIC, DIV_NUMERIC}

- **The remaining functions are signal processing functions that operate on time-domain signals (the terminals in T). The function set, F, for the result-producing branch and any automatically defined functions (except the arithmetic-performing subtrees) is**

F = {GAIN, INVERTER, LEAD, LAG, LAG2, DIFFERENTIAL_INPUT_INTEGRATOR, DIFFERENTIATOR, ADD_SIGNAL, SUB_SIGNAL, ADD_3_SIGNAL, ADF0, ADF1, ADF2, ADF3, ADF4}.

- **The two-argument DIV_NUMERIC function is never allowed to exceed 10^5.**

FREE VARIABLE — CONTINUED

- The fitness of a controller is measured using 42 elements, including
  (1) 40 time-domain-based elements based on a modified integral of time-weighted absolute error (ITAE) measuring how quickly the controller causes the plant to reach the reference signal, the robustness of the controller in face of significant variations in the plant's internal gain, *K*, the success of the controller in avoiding overshoot, and the effect of the externally supplied value of the time constant of τ,
  (2) one frequency-domain-based element measuring bandwidth, and
  (3) one frequency-domain-based element measuring the effect of sensor noise.

The fitness of an individual controller is the sum of these 42 elements of the fitness measure. The smaller the sum, the better.

FREE VARIABLE — CONTINUED

- **The first 40 elements of the fitness measure together represent**
  - **five different externally supplied values for the plant's time constant τ,**
  - **in conjunction with two choices of different values for the plant's internal gain, K,**
  - **in conjunction with two choices of different values for the height of the reference signal, and**
  - **in conjunction with two choices of different values of the variation in time constant τ, around its externally supplied value.**
- **The five values of τ, range between 0.1, 0.3, 1.0, 3.0, and 10.0.**
- **The two values of the plant's internal gain, K, are1.0 and 2.0 (for robustness in the face of variation in K)**
- **The reference signal rises from 0 to a specified height at t = 100 milliseconds. The two values for the height are 1 volt and 1 microvolts.**

FREE VARIABLE — CONTINUED

- **For each of these 40 elements of the fitness measure, a transient analysis is performed in the time domain using the SPICE simulator. The contribution to fitness for each of these 40 elements of the fitness measure is based on the integral of time-weighted absolute error .**
- **The integral is approximated from $t = 0$ to $t = 10\tau$, where τ is the externally supplied value of the time constant, τ.**
- **We multiply each value of the error $e(t)$ by the reciprocal of the amplitude of the reference signals so that both reference signals are equally influential.**

FREE VARIABLE — CONTINUED

- **In addition, we multiply each value of $e(t)$ by an additional weight, A, that varies depending on $e(t)$ and that heavily penalizes non-compliant amounts of overshoot. The function A weights all variations below the reference signal and all variations up to 2% above the reference signal by a factor of 1.0, but A heavily penalizes overshoots above 2% by a factor 10.0.**
- **Finally, we divide each value of the integral by τ^2 so as to equalize the influence of each of the five externally supplied values of τ.**

FREE VARIABLE — CONTINUED

- **The 41st element of the fitness measure is designed to constrain the frequency of the control variable so as to avoid extreme high frequencies in the demands placed upon the plant. This element of the fitness measure is based on 121 fitness cases representing 121 frequencies. Specifically, SPICE is instructed to perform an AC sweep of the reference signal over 20 sampled frequencies (equally spaced on a logarithmic scale) in each of six decades of frequency between 0.01 Hz and 10,000 Hz. A gain of less than 6 dB is acceptable for frequencies between 0.01 Hz and 60 Hz and a gain of less than -80 dB is acceptable for frequencies between 60 Hz and 10,000 Hz. For each of the 121 frequencies, the contribution to fitness is 0 if the gain is acceptable, but 1,000/121 otherwise.**

FREE VARIABLE — CONTINUED

- **The 42nd element of the fitness measure is based on 121 fitness cases representing 121 frequencies. SPICE is instructed to perform an AC sweep of the signal resulting by adding the plant response to a noise source ranging over 20 sampled frequencies (equally spaced on a logarithmic scale) in each of six decades of frequency between 0.01 Hz and 10,000 Hz. A gain of less than 6 dB is acceptable for frequencies between 0.01 Hz and 60 Hz and a gain of less than -80 dB is acceptable for frequencies between 60 Hz and 10,000 Hz. For each of the 121 frequencies, the contribution to fitness is 0 if the gain is acceptable but 1,000/121 otherwise.**
- **A controller that cannot be simulated by SPICE is assigned a high penalty value of fitness (10^8).**
- **Control Parameters: Population is 500,000.**

FREE VARIABLE — RESULTS

- The best-of-generation circuit from generation 0 has a fitness of 3,448.8.
- The best-of-run controller (from generation 42) has an overall fitness of 38.72. Notice that the free variable, τ, appears in five blocks of this controller.

FREE VARIABLE — RESULTS — CONT

• After simplification, it can be seen that the best-of-run is a PID-D2 controller with setpoint weighting followed by a simple first-order lag block.

• The four PID-D2 coefficients in the numerator are K_p = 17.0 for the proportional (P) term, $K_i = 6/\tau$ for the integrative (I) term, $K_d = 16\tau$ for the derivative (D) term, and $K_{d2} = 5\tau^2$ for the second derivative (D2) term.

• The coefficient for the setpoint weighting of the proportional term is b = 0.706. The coefficient for the setpoint weighting of the derivative term is c = 0.375. The coefficient for the setpoint weighting of the second derivative term is e = 0.0. The time-constant of the lag following the PID-D2 controller is $n = 0.5\tau$.

FREE VARIABLE — RESULTS — CONT

COMPARISON OF THE TIME-DOMAIN RESPONSE OF THE BEST-OF-RUN GENETICALLY EVOLVED CONTROLLER (TRIANGLES) FROM GENERATION 42 AND THE ASTROM AND HAGGLUND CONTROLLER (SQUARES) WITH *K* = 1 AND τ = 1

FREE VARIABLE — RESULTS — CONT

COMPARISON OF THE TIME-DOMAIN RESPONSE OF THE GENETICALLY EVOLVED CONTROLLER (TRIANGLES) FROM GENERATION 42 AND THE ASTROM AND HAGGLUND CONTROLLER (SQUARES) TO A 1-VOLT DISTURBANCE SIGNAL WITH K = 1 AND τ = 1

FREE VARIABLE — RESULTS — CONT

COMPARISON FOR K = 1.0, τ = 1.0, AND STEP SIZE OF 1.0.

	Units	GP controller	Astrom and Hagglund 1995
ITAE for unit-step reference signal	volt sec^2	0.77	2.84
Rise time	seconds	1.43	2.51
Settling time	seconds	4.66	8.50
Disturbance sensitivity	volts/volts	0.07	0.19
IATE for unit-step disturbance	volt sec^2	0.42	1.35
Noise Attenuation Corner Frequency	Hz	0.72	0.34
Noise Attenuation Roll Off	dB/Decade	40	40
Maximum Sensitivity	volts/volts	2.09	2.11

- **GP controller is better than 3.69 times as effective as the Astrom and Hagglund 1995 controller as measured by ITAE, has only 57% of the rise time in response to the reference input, and has only 55% of the settling time.**

FREE VARIABLE — RESULTS — CONT

• Suppose $G(s)$ is the transfer function of a plant and we wanted to change all the time constants of this plant by the same amount, say τ. To do this, we would produce a plant with transfer function, $G_\tau(s) = G(\tau s)$. In this new transfer function $G_\tau(s)$, each coefficient of s would be multiplied by τ; each coefficient of s^2 would be multiplied by τ^2; and each coefficient of $1/s$ would be multiplied by $1/\tau$.

• A human designer might approach this problem as follows: Call the plant with $\tau = 1$ the reference plant. Design a controller for this reference plant satisfying the design requirements and call the new controller, H, the reference controller. Call the controlled plant, J, the reference controlled plant. Let $H(s)$ be the transfer function of controller $H(s)$, and let $J(s)$ be the transfer function of the controlled plant.

FREE VARIABLE — RESULTS — CONT

• Now, to handle the time scaling, let $H_\tau(s)$ = $H(\tau s)$ be the controller design for the time-scaled plant. This will produce an entire controlled plant, $J_\tau(s)$, scaled by the same factor τ; J_τ (s) = $J(\tau * s)$. Effectively all the behavior of the time-scaled controlled plant is that of the reference controlled plant with the time scale changed.

• In fact, we did exactly this in order to produce a PID controller with set point weighting for comparison purposes above (using the design rules using M_s = 2 on page 225 of Astrom and Hagglund 1995).

FREE VARIABLE — RESULTS — CONT

• Referring now to genetically evolved controller shown above, notice that we could have written the following four coefficients in the control signal, $U(s)$, as follows:

$K_i/s = 6/\tau s$

$K_d\, s = 16\tau s^2$

$K_{d2}\, s^2 = 5\tau^2 s^2$

$ns = 0.5\tau s$

• This is, in fact, the very same change in time scale as the human designed would have used.

FREE VARIABLE — RESULTS

BEST-OF-RUN CONTROLLER FROM GENERATION 42

Notice that the free variable, τ, appears in five blocks of this controller

COMMON PARAMETERIZED CONTROLLER FOR TWO FAMILIES OF PLANTS

- **Family of *n*-lag plants (n = 3, 4, and 8)**

- **Family of plants (α = 0.2, 0.5, and 0.7)**

COMMON PARAMETERIZED CONTROLLER FOR TWO FAMILIES OF PLANTS

- The numerical parameter value for each signal processing block possessing a parameter is established by an arithmetic-performing subtree
- The terminal set, T_{aps}, for the arithmetic-performing subtrees is

$T_{aps} = \{\Re$, L, TR, KU, TU$\}$

  - $\Re$ — perturbable numerical values (from 10^{-3} and 10^{3})
  - T_r — time constant
  - L — dead time
  - K_u — ultimate gain — the minimum value of the gain in the feedback path to cause a system to oscillate
  - T_u — ultimate period — the period of this lowest frequency oscillation
- The function set, F_{aps}, for the arithmetic-performing subtrees is

F_{aps} = {`ADD_NUMERIC`, `SUB_NUMERIC`, `MUL_NUMERIC`, `DIV_NUMERIC`, `REXP`, `RLOG`}

OVERALL MODEL

COMMON PARAMETERIZED CONTROLLER FOR TWO FAMILIES OF PLANTS

COEFFICIENTS OF PID CONTROLLER EQUIVALENT TO THE BEST-OF-RUN CONTROLLER FROM GENERATION 217