Problem Set No. 1
BMI 226 / CS 426
Spring 2003
NAME $\qquad$
(1) Goldberg GASOML - Ch 1 - problem 1.1
(2) Goldberg GASOML - Ch 1 - problem 1.2
(3) Goldberg GASOML - Ch 1 - problem 1.3. Be sure to mention the result of the comparison.
(4) Goldberg GASOML - Ch 1 - problem 1.4
(5) Goldberg GASOML - Ch 1 - problem 1.9
(6) Goldberg GASOML - Ch 1 - problem 1.10
(7) Goldberg GASOML - Ch 2 - problem 2.1
(8) Goldberg GASOML - Ch 2 - problem 2.2
(9) Combinatorics: Fill in with an unexpanded mathematical expression for strings of length $L$ in a population of size M .
(a) For a binary alphabet (i.e. $\mathrm{K}=2$ ), there are $\qquad$ schema.
(b) For an alphabet with K characters, there are $\qquad$ schema.
(c) The specificity (i.e., order) $\mathrm{O}(\mathrm{H})$ of a schema H can vary between $\qquad$ and $\qquad$ .
(d) The defining length $\delta(\mathrm{H})$ of a schema H can vary between $\qquad$ and $\qquad$ .
(e) For a binary alphabet, there are $\qquad$ individual points in a schema of order $\mathrm{O}(\mathrm{H})$.
(f) For a binary alphabet, these points can be viewed as a hyperplane of dimensionality
$\qquad$ .
(g) For a binary alphabet, there are $\qquad$ such hyperplanes of dimensionality $i$, where i varies from 0 to L .
(h) For a binary alphabet $(\mathrm{K}=2)$, a particular individual string of length L is represented in schema.
(i) For a string over an alphabet of size K , a particular individual string of length L is represented in $\qquad$ schema.
(j) In how many ways can a single mutation operation be performed in a population of size $M$ of strings of length $L$ ?
(k) In how many ways can a single crossover operation be performed in a population of size M of strings of length L?
(L) How many different initial random populations are there in a population of size M of strings of length L with an alphabet of size K ?
(m) What is the expected number of occurrences of an arbitrary schema of order $\mathrm{O}(\mathrm{H})$ in the initial random population (generation 0 ) of a run of genetic programming?
(n) How many schema, in total, are represented in a population of size $M$ of strings of length $L$ with an alphabet of size K?
(10) Different Kind of Crossover: The conventional crossover operation selects one of L-1 interstitial places between the L characters of a chromosome string as the crossover point. Consider a new crossover operation in which an arbitrary subset of the L positions is selected. The characters of the first binary string are replaced by the positions of the second binary string belonging to a certain selected subset of the original L positions in order to obtain the first offspring. The second offspring for this newly defined crossover operation is obtained in a symmetric way.
(a) The conventional crossover can be performed in L-1 ways. How many ways can this new crossover be performed?
(b) What is the expected number of schema $m(H, t+1)$ in terms of $m(H, t)$ for this new crossover operation?
(11) Fourier Series: The figure shows the function $p(x)=x^{2}$ over the interval $[-\pi,+\pi]$. In addition, the function from that limited interval is repeated over one subsequent and one preceding interval of width of $2 \pi$.


A function such as this can be represented by the infinite Fourier series
$p(x)=\mathbf{a}_{0}+\sum_{\mathrm{j}=1}^{\infty} \mathrm{a}_{\mathrm{j}} \operatorname{Cos} \mathrm{j} \boldsymbol{q}+\mathrm{b}_{\mathrm{j}} \operatorname{Sin} \mathrm{j} \boldsymbol{q}$
The first few terms of the Fourier series for this particular function $x^{2}$ are
$\mathrm{p}(\mathrm{x})=\boldsymbol{x}^{2}=\pi / 3+0.0 \operatorname{Sin} x-4 \operatorname{Cos} x$
$+0.0 \operatorname{Sin} 2 x+\frac{4 \operatorname{Cos} 2 x}{2^{2}}+0.0 \operatorname{Sin} 3 x-\frac{4 \operatorname{Cos} 3 x}{3^{2}}$
$+0.0 \operatorname{Sin} 4 x+\frac{4 \operatorname{Cos} 4 x}{4^{2}}-\ldots$
Suppose we want to use the genetic algorithm operating on a fixed-length character string to find the Fourier coefficients of the first nine terms of the Fourier series for the $p(x)$ function. The $p(x)$ function is a black box which you are free to interrogate to find its value for any value of the independent variable $x$. NOTE: You do not have foreknowledge that the function is even (i.e., the coefficients of the sine terms are all zero). Please fill in the tableau below for applying the GA to this problem.

| Objective: |  |
| :--- | :--- |
| Representation scheme: | $\bullet$$\bullet$ Structure $=$ <br>  <br> $\bullet \mathrm{K}=$ <br> $\bullet \mathrm{L}=$ <br> $\bullet$ Mapping $=$ |
| Fitness cases: |  |
| Fitness: |  |
| Parameters: | $\bullet M=$ |
| Termination criteria: |  |
| Result designation: |  |

